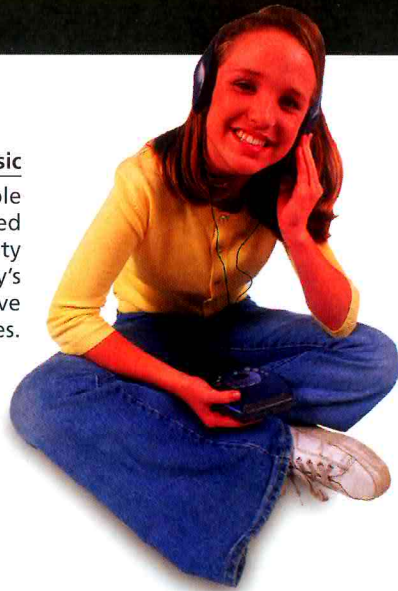### Portable Music

Early models of portable CD players often skipped or had poor quality headphones. Today's models skip less and have better headphones.

Musical Interlude

A teenager spends about $9\frac{1}{2}$ h/wk listening to music and buys about $350 worth of CDs a year.

The center area contains no music.

This area contains music.

Take It to the NET For more information about music, go to **www.PHSchool.com**.
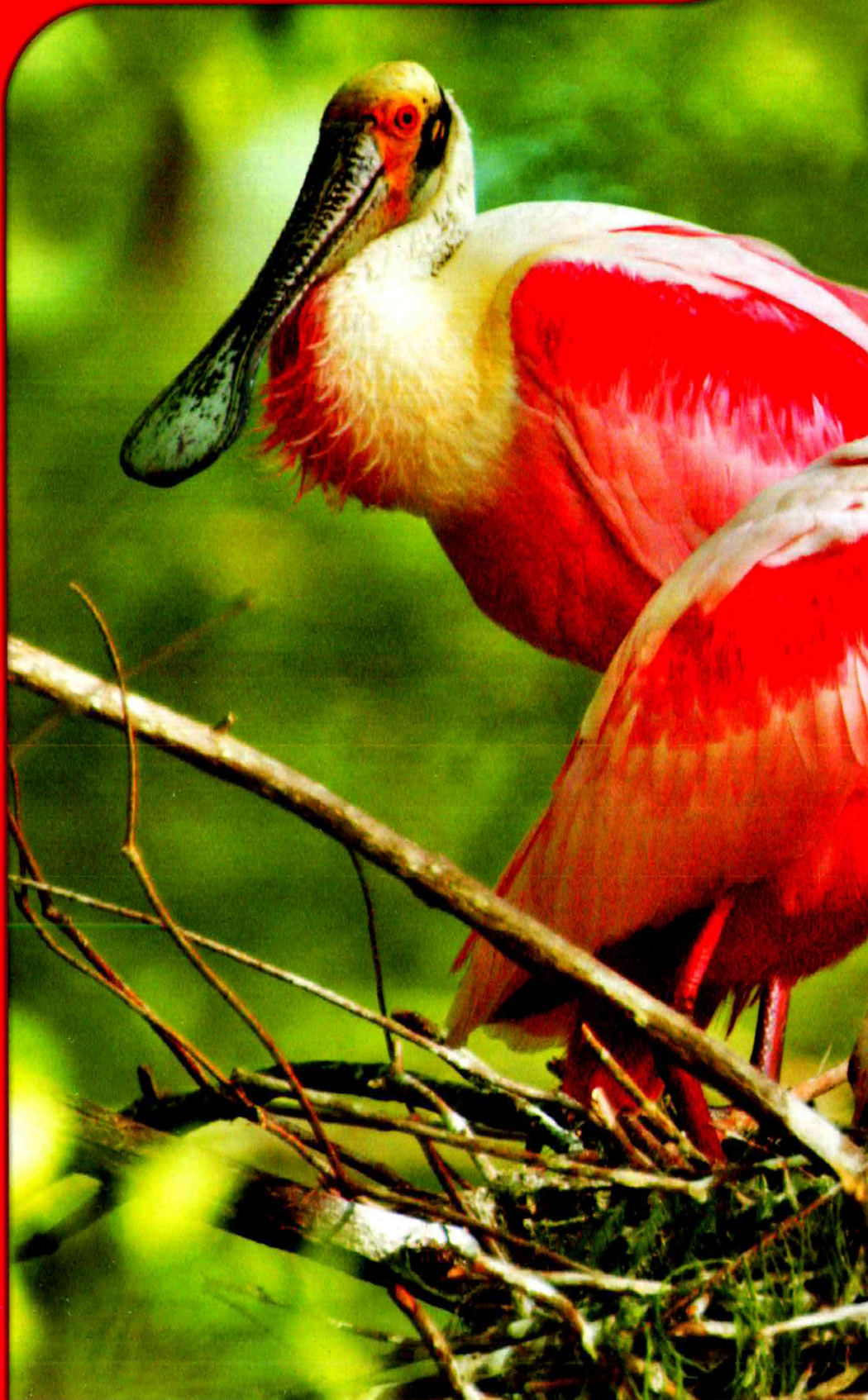Web Code: abe-0853

CHAPTER 9

Patterns and Rules

Key Vocabulary

- arithmetic sequence (p. 474)

- balance (p. 500)

- compound interest (p. 500)

- conjecture (p. 475)

- formula (p. 507)

- function (p. 484)

- geometric sequence (p. 475)

- principal (p. 499)

- sequence (p. 474)

- simple interest (p. 499)

Real-World Snapshots

Roseate spoonbills, or "pinks" as they are sometimes called, live as long as 10 years in captivity. Now protected, spoonbills were once hunted for their brilliant feathers.

Data File Animal Longevity

Animal	Birth Weight (Male) (g)	Maximum Life Span (yr)
Dragonfly	1	0.1
Rat	6	3.3
Salmon	15	13
Spoonbill	45	10
Rabbit	65	13
Cat	98	28
Dog (cocker spaniel)	240	20

You will use the data above in this chapter:

Real-World Snapshots On pages 516 and 517, you will solve problems involving animal longevity.

Chapter 9 Preview

Where You've Been

- In Chapter 2, you wrote algebraic expressions and equations to represent patterns and real-world situations. You also solved equations.

- In Chapter 5, you used proportions to represent patterns.

Where You're Going

- In Chapter 9, you will represent patterns by using function tables, writing rules, and drawing graphs.

- Applying what you learn, you will write and graph a function rule to find distance traveled over time.

Pilots use functions to plan flights.

iTEXT Instant self-check online and on CD-ROM

Diagnosing Readiness

? For help, go to the lesson in green.

Evaluating and Writing Algebraic Expressions (Lesson 2-1)

Evaluate each expression using $r = 4, s = -2,$ and $t = 5.1$.

1. $3r - t$ **2.** rst **3.** $8s^2 + rt$ **4.** $1.5(1 + s)^r$

Solving Two-Step Equations (Lesson 2-6)

Solve each equation.

5. $3x - 1 = 14$ **6.** $10 + 3n = 25$ **7.** $4(b - 3) = 7$ **8.** $-5x = 3x - 22$

Exponents and Order of Operations (Lesson 3-1)

Simplify.

9. $2^3 \cdot 2 - 4^2$ **10.** $2^3 \cdot (2 - 4)^2$ **11.** $(3 - 2)^2 - 2^2$ **12.** $4^3 + 4 \div 4^2$

Solving Equations With Fractions (Lesson 4-6)

Solve each equation.

13. $\frac{2}{3}n - 10 = 14$ **14.** $\frac{x}{7} = 49$ **15.** $1.5 + \frac{4}{5}a = 21$ **16.** $\frac{1}{2}p + \frac{1}{6} = 1\frac{1}{2}$

Percents, Fractions, and Decimals (Lesson 6-2)

Write each percent as a decimal.

17. 4%   **18.** 12%   **19.** 3.58%   **20.** 4.05%   **21.** 10.3%

Patterns and Graphs

What You'll Learn

OBJECTIVE 1 To make graphs

OBJECTIVE 2 To use graphs to make estimates

. . . And Why

To find the cost of a phone call, as in Example 3

✔ Check Skills You'll Need

For help, go to Lesson 3-10.

Draw a number line from 0 to 10. Graph and label each point.

1. 7

2. 9.5

3. $3\frac{1}{2}$

4. 2.3

5. 0.8

6. $4\frac{3}{4}$

7. Order from least to greatest.

$$-5.6, 3\frac{1}{3}, -\frac{5}{6}, 2.98, -5.\overline{5}$$

OBJECTIVE 1 Making Graphs

📱 **iTEXT** Interactive lesson includes instant self-check, tutorials, and activities.

Graphs can help you see patterns in data. A graph includes two *scales*, or rulers—the horizontal axis and the vertical axis. An *interval* is the difference between the values on a scale.

1 EXAMPLE Choosing Scales and Intervals

Graph the data in the table at the left.

Interest on Savings

Amount Saved ($)	Interest Earned ($)
200	8
350	14
500	20
750	30
900	36

Step 1 Choose the scales and intervals.

Use the horizontal scale of the graph for the data in the first column. Use the vertical scale for the data in the second column. Start both scales at 0.

- Choose the interval for the horizontal scale. The greatest amount saved is $900. If each interval is $100, then the number of intervals is 900 ÷ 100, or 9.
- Choose the interval for the vertical scale. The greatest interest earned is $36. If each interval is $4, then the number of intervals is 36 ÷ 4, or 9.

Interest on Savings
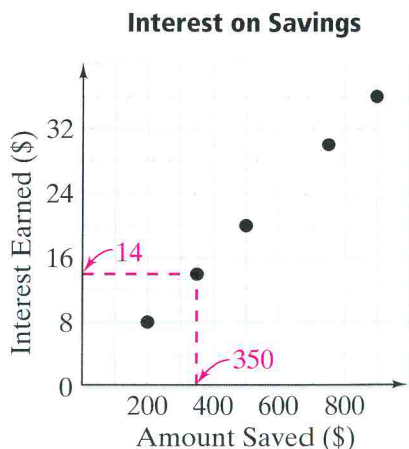
Step 2 Use points to represent the data. The red dashes show how to plot the point representing Interest Earned of $14 on an Amount Saved of $350.

✔ Check Understanding 1

a. Graph the data in Example 1 using a vertical interval of $5.

b. **Reasoning** Which interval is easier to graph—$4 or $5? Explain.

Graphs having from 6 to 10 intervals are easy to read.

2 EXAMPLE Real-World Problem Solving

Stuffed Animals

Number	Cost ($)
100	40
200	67
300	94
400	121
500	148

Toys A toy manufacturer produces stuffed animals. Graph the manufacturing costs.
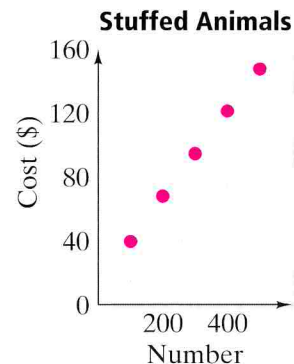
The pattern in the first column of data suggests a horizontal interval of 100.

The greatest value in the second column is $148.

$148 ÷ 10 intervals, or $14.80 per interval
$148 ÷ 6 intervals, or $24.67 per interval

Choose a vertical interval that is between $14.80 and $24.67. An interval of $20 is easy to use.

• Use points to represent the data.

Stuffed Animals

✔ **Check Understanding** ② Graph the data in the table at the right.

Yogurt Costs

Yogurt (pt)	Price ($)
50	26
100	49
150	72
200	95

OBJECTIVE

2 Using Graphs to Make Estimates

You can use a graph to make estimates between data points.

3 EXAMPLE Estimating on a Graph Real World

Cellular Phones The graph below shows the costs of different cell phone calls. The points are connected because the phone company charges for the exact amount of time a call takes. How much does a call of $3\frac{1}{2}$ min cost?

Cell Phone Cost
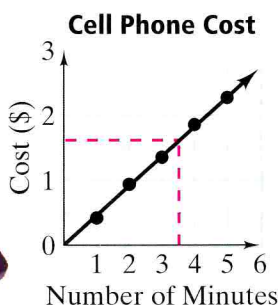
Draw lines to locate the value on the vertical axis that corresponds to $3\frac{1}{2}$ on the horizontal axis.

The cost is greater than $1.50, but less than halfway between $1.50 and $2.00, or $1.75. Estimate the answer as $1.60.

• The cost of a $3\frac{1}{2}$-min call is about $1.60.

✔ **Check Understanding** ③ Use the graph in Example 3 to estimate the cost of a $4\frac{1}{2}$-min call.

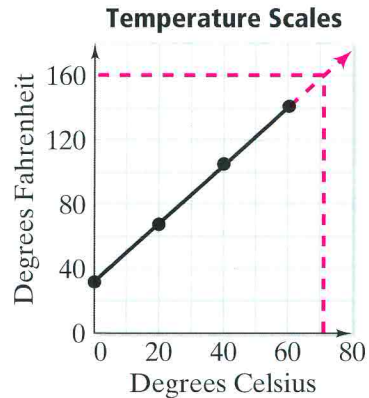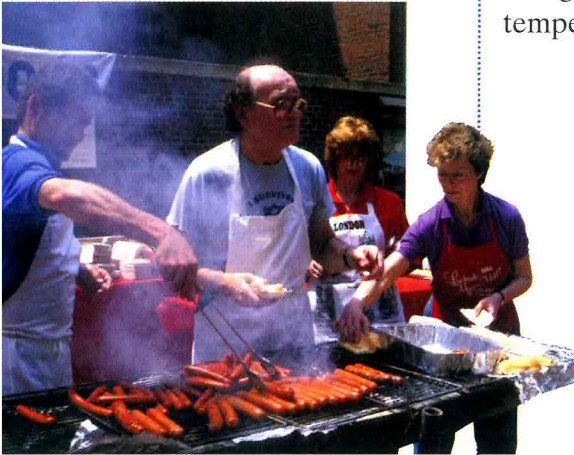You can use a graph to make a prediction. Extend the graph and find a corresponding value on the appropriate axis.

④ **EXAMPLE**   **Extending a Graph**

The graph shows the relationship between Celsius and Fahrenheit temperatures. Estimate the Celsius temperature for 160°F.

Temperature Scales

← Extend the graph as shown.

For 160°F, the Celsius temperature
← is slightly more than 70°. Estimate the answer.

A temperature of 160°F is about 71°C.

✔ **Check Understanding** ④ **a.** Estimate the Fahrenheit temperature for 80°C.
b. **Reasoning** For what type of data would you need to extend the graph in the opposite direction?

EXERCISES

❓ For more practice, see *Extra Practice.*

Ⓐ **Practice by Example**   **Graph the data in each table.**

Example 1
(page 469)

1.   **Shoveling Snow**

Hours Worked	Salary ($)
2	18
4	36
5	45
6	54

2.   **A Cow's Weight**

Age (months)	Weight (lb)
1	15
2	28
3	40
4	54
5	66

3. Use a different interval for the vertical axis to graph the data from Exercise 2.

Example 2
(page 470)

Graph the data in each table.

4.   **Plant Growth**

Age (yr)	Height (cm)
5	90
7	95
9	102
11	110

5. **Used Dirt Bike Prices**

| Age (yr)	Price ($)
2	43
4	37
6	30

Example 3
(page 470)

Estimate using your graphs from Exercises 4 and 5.

6. the height of an 8-year-old plant

7. the plant's age when it was 105 cm tall

8. the price of a used bike that is 5 years old

9. the age of a bike that is being sold for $40

Example 4
(page 471)

Estimate using the graph at the right.

Electricity Charges

10. the cost of 85 h of electricity

11. the cost of 100 h of electricity

12. How many hours cost $6.50?
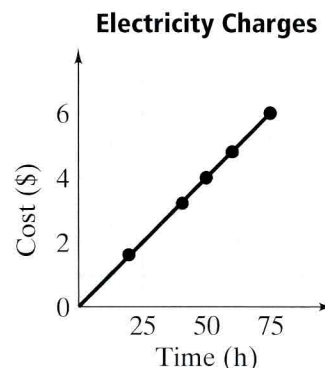
B **Apply Your Skills**

13. a. **Geometry** Graph the perimeters of
      squares with side lengths of
      1, 2, 3, 4, and 5 in.
   b. Estimate the side length of a square
      with perimeter 9.6 in.
   c. Estimate the perimeter of a square with side length 3.5 in.
   🖩 d. **Calculator** Test your estimates with a calculator. Were your
      estimates correct?

14. **Physical Fitness** To qualify for the Presidential
    Physical Fitness Award, girls between the ages of
    11 and 14 must be able to run one mile within
    the times listed.
   a. Graph the data.
   b. Estimate the time for a 17-year-old girl.
   c. **Error Analysis** Why might your estimate be
      inaccurate?

Age (yr)	Time (mins)
11	9.03
12	8.38
13	8.22
14	7.98

Real-World 🌎 Connection

The Presidential Physical
Fitness Challenge consists of
five fitness tests, including
pull-ups.

Graph the data in each table. Use your graph to estimate the missing value.

15.
Hours of Sleep	Math Test Score
9	93
8	85
7	74
6	n

16.
Age (yr)	Weight (lb)
1	3.5
2	n
4	8
5	9.5

17.
Time (h)	Temp. (°C)
1	12
2	15
5	24
8	n

18.
Time (s)	Distance (mi)
10	15
15	22.5
25	37.5
n	48

19. **Writing in Math** Describe what a graph looks like when both sets of
    values increase.

20. Data file, p. 467 Draw a graph of the animal longevity data. Place Life Span on the vertical axis and Birth Weight on the horizontal axis.

C **Challenge** 🌐 **21. Books** The table shows costs to print books.

a. Estimate the cost of 2,500 books.
b. Estimate the cost of 7,500 books.
c. Find the unit costs to print 5,000 and 10,000 books. Why does the unit cost depend on the size of the order?

Number of Books	Cost ($)
5,000	175,000
10,000	290,000

🌐 **22. Work** Suppose a neighbor will pay you $10 per week to wash windows. Another neighbor will give you $40 and then pay $7 per week.

a. Make two tables, one for each neighbor, showing the amount you receive from each neighbor for 1, 2, 3, 4, and 5 weeks of work.
b. Graph both sets of data on the same axes.
c. How much will you receive from each neighbor after 10 weeks?
d. **Reasoning** Which method of payment do you prefer? Explain.

23. Stretch Your Thinking How many ways can you arrange 10 blocks in 3 containers so that each has an even, nonzero number of blocks?

Test Prep

Multiple Choice

For Exercises 24 and 25, graph the data at the right.

24. Estimate the value of y when $x = 12$.
  A. 12   B. 16   C. 24   D. 30

25. Estimate the value of x when $y = 18$.
  F. 10   G. 14   H. 28   I. 30

x	y
-1	3
0	4
2	6
4	8

Extended Response

26. a. Graph the data for bacteria growth.
  b. When the environment changes, the growth rate changes. Use your graph to estimate when a change in the environment occurred.
  c. Estimate the time at which there were 12 bacteria.

Bacteria Growth

Time (min)	Number of Bacteria
0	5
20	10
40	15
60	20
80	60

Take It to the NET
Online lesson quiz at
www.PHSchool.com
Web Code: aba-0901

Mixed Review

Lesson 6-2 **Write each percent as a decimal.**

27. 38%   **28.** 5%   **29.** 150%   **30.** 6.2%

Lesson 1-7 **Find each sum or difference.**

31. $-4 + 7$ **32.** $-4 - 7$ **33.** $-4 + (-7)$ **34.** $-4 - (-7)$

9-2 Number Sequences

What You'll Learn

OBJECTIVE 1 To use arithmetic sequences

OBJECTIVE 2 To use geometric and other sequences

. . . And Why

To identify types of patterns, as in Example 3

✔ Check Skills You'll Need

For help, go to Lesson 1-7.

Add.

1. $-3 + 3$ 2. $-3 + 2$
3. $-3 + 1$ 4. $-3 + 0$
5. $-3 + (-1)$ 6. $-3 + (-2)$

New Vocabulary
- sequence
- arithmetic sequence
- geometric sequence
- conjecture

OBJECTIVE 1

Using Arithmetic Sequences

i**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

Investigation: Finding a Pattern

Use pattern blocks or draw diagrams.

1. Make the next two figures in the pattern below.
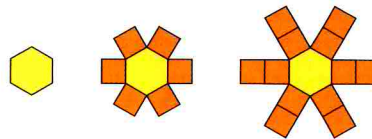
Figure 1   Figure 2   Figure 3

2. How many blocks do you add to each figure to make the next figure in the pattern?

3. Copy and complete the table below.

Figure	1	2	3	4	5	6	7	8
Total Blocks in Figure	1							

4. Describe any patterns that you notice in your table.

The set of numbers $1, 3, 5, 7, 9, \ldots$ has a pattern. If you add 2 to any number, you get the next number in the set.

A **sequence** is a set of numbers that follow a pattern. Each number in a sequence is a *term*. You can find each term of an **arithmetic sequence** by adding a fixed number (called the common difference) to the previous term.

Reading Math

You pronounce *arithmetic sequence* as "ar-ith-MEH-tik SEE-kwens."

1 EXAMPLE — Rules for Arithmetic Sequences

Write a rule to describe the sequence $12, 7, 2, -3, \ldots$. Then find the next three terms in the sequence.

$$12 \quad\searrow\nearrow\quad 7 \quad\searrow\nearrow\quad 2 \quad\searrow\nearrow\quad -3$$

$$+(-5) \qquad +(-5) \qquad +(-5) \qquad \leftarrow \text{Find the common difference.}$$

The rule is *Start with 12 and add* -5 *repeatedly.*
The next three terms are $-8, -13,$ and -18.

✔ **Check Understanding** 1 a. Write a rule for the sequence $44, 35, 26, 17, \ldots$. Find the next three terms.

b. **Reasoning** The rule for a sequence is *Start with 21 and subtract 11 repeatedly.* Is it an arithmetic sequence? Explain.

OBJECTIVE

2 Using Geometric and Other Sequences

In a **geometric sequence,** you find each term by multiplying the previous term by a fixed number (called the common ratio).

2 EXAMPLE — Rules for Geometric Sequences

Write a rule for the sequence $27, 9, 3, 1, \ldots$. Find the next three terms.

$$27 \quad\searrow\nearrow\quad 9 \quad\searrow\nearrow\quad 3 \quad\searrow\nearrow\quad 1$$

$$\cdot \frac{1}{3} \qquad \cdot \frac{1}{3} \qquad \cdot \frac{1}{3} \qquad \leftarrow \text{Find the common ratio.}$$

The rule is *Start with 27 and multiply by* $\frac{1}{3}$ *repeatedly.*

$$\left.\begin{array}{l} 1 \cdot \frac{1}{3} = \frac{1}{3} \\[4pt] \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \\[4pt] \frac{1}{9} \cdot \frac{1}{3} = \frac{1}{27} \end{array}\right\} \leftarrow \text{Find the next three terms.}$$

The next three terms are $\frac{1}{3}, \frac{1}{9},$ and $\frac{1}{27}$.

Need Help?

Dividing is the same as multiplying by a reciprocal.

✔ **Check Understanding** 2 a. Write a rule for the sequence $1,000; 100; 10; \ldots$. Find the next three terms.

b. **Reasoning** The rule for a sequence is *Start with 12 and divide by* -4 *repeatedly.* Is it a geometric sequence? Explain.

A sequence can be arithmetic, geometric, both, or neither. It is neither arithmetic nor geometric if you cannot find a common difference or a common ratio.

A **conjecture** is a prediction that suggests what you expect will happen. When you look for a pattern and write a rule to describe the pattern in a sequence, you are using *inductive reasoning*. The conjectures you make may not always be true. Check your results whenever possible.

3 **EXAMPLE**   **Real-World** **Problem Solving**

Geometry Write a rule to find the
number of circles in each figure.
Is the sequence *arithmetic, geometric,*
both, or *neither*?
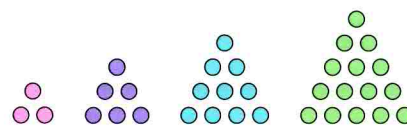
$$3 \qquad 6 \qquad 10 \qquad 15\dots \quad \leftarrow \text{number of circles in each figure}$$

$$\searrow \nearrow \searrow \nearrow \searrow \nearrow$$

$$+3 \qquad +4 \qquad +5 \quad \leftarrow \text{Look for a common difference or a common ratio.}$$

The rule is *Start with 3 and add consecutive integers. First add 3, then*
add 4, and so on.

The sequence is neither arithmetic nor geometric.   ← conjecture

Look Back and Check Is there a common ratio?

$$3 \qquad 6 \qquad 10 \qquad 15$$

$$\searrow \nearrow \searrow \nearrow \searrow \nearrow$$

$$\cdot 2 \qquad \cdot 1\frac{2}{3} \qquad \cdot 1\frac{1}{2}$$

You cannot multiply by or add the same number to each
← term to find the next term. The sequence is neither
arithmetic nor geometric. The conjecture is correct.

✔ **Check Understanding** **3** Identify each sequence as *arithmetic, geometric, both,* or *neither.*
   a. $1, 2, 6, 24, \dots$ **b.** $2, 3, 6, 11, \dots$ **c.** $10, 9, 8, 7, \dots$

EXERCISES

❓ For more practice, see *Extra Practice.*

A **Practice by Example**

Example 1
(page 475)

Example 2
(page 475)

Write a rule for each sequence. Then find the next three terms.

1. $5, 10, 15, 20, \dots$ **2.** $3, 7, 11, 15, \dots$ **3.** $34, 29, 24, 19, \dots$

4. $25, 21, 17, 13, \dots$ **5.** $63, 54, 45, 36, \dots$ **6.** $-8, -1, 6, 13, \dots$

7. $1, 2, 4, 8, \dots$ **8.** $2, -6, 18, -54, \dots$ **9.** $600, -300, 150, \dots$

10. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$ **11.** $-2, 4, -8, 16, \dots$ **12.** $\frac{1}{4}, \frac{1}{12}, \frac{1}{36}, \frac{1}{108}, \dots$

Example 3
(page 476)

Identify each sequence as *arithmetic, geometric, both,* or *neither.*

13. $2, 5, 10, 17, 26, \dots$ **14.** $1, 4, 9, 16, 25, \dots$ **15.** $7, 14, 28, 56, \dots$

16. $-2, -2, -2, \dots$ **17.** $300, 60, 12, 2.4, \dots$ **18.** $84, 63, 42, 21, \dots$

🌎 **19. Employment** Suppose an employer pays new employees $8/h the first
   year, $9/h the second year, $10/h the third year, and $15/h the fourth
   year. Is the pattern *arithmetic, geometric, both,* or *neither*?

B **Apply Your Skills** 🌎 **20. Running** Mario can run a mile in 9 min. After 4 months of training for a
   marathon, he hopes to run a mile in 8 min. His time decreases 15 s each
   month. What would you tell Mario about his conjecture?

21. **Science** A female bee has two biological parents—a female and a male. A male bee has only one parent, a female.
    a. Make a family tree that shows seven generations of ancestors of a male bee. Find the number of ancestors in each.
    b. The numbers of ancestors form a number sequence. Is it *arithmetic*, *geometric*, *both*, or *neither*?

Ancestors
3
2
1

Write a rule for each sequence. Then find the next three terms.

22. $2, 4, 8, 16, \ldots$ 23. $4, -8, 16, -32 \ldots$ 24. $2, -4, 8, -16, \ldots$

25. $-2, -4, -8, -16, \ldots$ 26. $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$ 27. $-5, -1, 3, 7, \ldots$

28. $1, 5, 14, 30, 55, \ldots$ 29. $1, 5, 13, 29, \ldots$ 30. $-1, 1, -2, 2, \ldots$

31. For Exercises 26–30, tell whether each sequence is *arithmetic*, *geometric*, *both*, or *neither*.

32. a. How many blue tiles will be in the ninth figure of the pattern? How many yellow tiles?
    b. Write a rule to describe the pattern.

33. **Pattern** Look at the pattern below and make a conjecture about the next term in the sequence. Test your conjecture with a calculator.

$123 \times 9 = 1,107$ $1,234 \times 9 = 11,106$ $12,345 \times 9 = 111,105$

34. **Nature** The Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ occurs in nature.
    a. Is the Fibonacci sequence *arithmetic*, *geometric*, *both*, or *neither*?
    b. Find the ninth and tenth terms in the Fibonacci sequence.
    c. **Reasoning** Is the sequence in Exercise 21 a Fibonacci sequence?

Write a rule for each sequence.

35. $1, 1.5, 2, 2.5, \ldots$ 36. $0.4, 0.8, 1.2, 1.6, \ldots$

37. $10, 9.3, 8.6, 7.9, \ldots$ 38. $0.1, 0.3, 0.9, 2.7, \ldots$

39. $0.5, 1.5, 4.5, 13.5, \ldots$ 40. $0.2, 0.4, 0.6, 0.8, \ldots$

41. the even integers greater than 12   42. the negative odd integers

43. a. **Geometry** Find the area of the rectangle.
    b. Double the dimensions and find the new area.
    c. Make a conjecture about how the area changes as the dimensions are doubled.
    d. Test your conjecture with a different rectangle.

3 cm
5 cm

44. **Writing in Math** Every term of a sequence is 1. Is it *arithmetic*, *geometric*, *both*, or *neither*? Explain.

45. a. List rows 4, 5, 6, and 7 in the pattern.
   b. Find the sum of each row.
   c. Write a rule for finding the sum of the numbers in each row.
   d. Reasoning Predict the sum of the numbers in the twentieth row.

row 1	1
row 2	1 2 1
row 3	1 2 3 2 1

**Calculator** Make a conjecture about the next term in each sequence. Test your conjecture with a calculator.

46. $2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = \blacksquare$

47. $3^4 = 81$
$3^3 = 27$
$3^2 = 9$
$3^1 = 3$
$3^0 = \blacksquare$

48. $4^4 = 256$
$4^3 = 64$
$4^2 = 16$
$4^1 = 4$
$4^0 = \blacksquare$

C Challenge

49. Make a conjecture about the next term in the sequence, and find it.
$1, 4, -1, 6, -3, 8, -5, 10, -7, 12, \ldots$

50. Geometry How does the volume of a rectangular solid change when its dimensions are doubled? Make a conjecture and test it.

51. Stretch Your Thinking When you draw either one of two diagonals in a polygon, it is divided into two congruent triangles. What is the polygon?
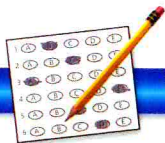
Test Prep

Multiple Choice

52. What are the next two terms in the sequence 1, 64, 2, 32, 4, 16, . . . ?
A. 16, 8   **B.** 8, 16   **C.** 8, 8   **D.** 32, 64

53. A rectangular ranch is 2 mi long by 6 mi wide. If its dimensions are doubled, by how many times will its area increase?
F. 2   **G.** 3   **H.** 4   **I.** 8

54. What is the next term in the sequence 1, -2, 4, -5, 7, -8, . . . ?
A. -10 **B.** 9   **C.** 10   **D.** 11

Short Response

55. The sum of two numbers is always greater than each of the numbers. Show why you think this statement is true or prove that it is false.

Mixed Review

Lesson 5-7

The scale on a map is 2 in. : 50 mi. Find the actual distance for each map distance. Round your answer to the nearest tenth.

**56.** 3 in.   **57.** $\frac{1}{2}$ in.   **58.** $1\frac{3}{4}$ in.   **59.** 5 in.

Lesson 5-1

Write each ratio in simplest form.

60. $\frac{200}{550}$ **61.** $\frac{270}{45}$ **62.** $\frac{18}{60}$ **63.** $\frac{24}{120}$

Read through the problem below. Then follow what Jacob thinks as he solves the problem. Check your understanding with the exercise at the bottom of the page.

a. **Geometry** Find the area of the rectangle.
b. Double the dimensions and find the new area.
c. Make a conjecture about how the area changes as the dimensions are doubled.
d. Test your conjecture with a different rectangle.

3 cm
5 cm
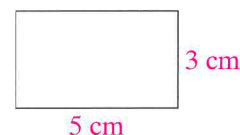
What Jacob Thinks

I'll write what I know from the diagram.

To find area, I can use the length and width.

Doubling the dimensions means doubling the length and width. Then I can find the area of the new rectangle.

To write a conjecture, I need to see a pattern. Sometimes a table helps me see a pattern in a set of numbers. I'll make a table.

I see the pattern. The new area is 4 times the original area. I can write a conjecture now.

To test my conjecture, I'll try a new rectangle by doubling the dimensions again.

So, my conjecture does work! The new area is four times the area of my original rectangle.

What Jacob Writes

Length = 5 cm   Width = 3 cm

a. $A = \ell w = 5 \cdot 3 = 15$
The area of the rectangle is 15 cm².

b. New length = $2 \cdot 5 = 10$ cm
New width = $2 \cdot 3 = 6$ cm

$A = 10 \cdot 6 = 60$ The new area is 60 cm².

c.

	w	Area
5	3	15
↓ · 2	↓ · 2	↓ · ?
10	6	60

← $15 \cdot 4 = 60$

Conjecture: When a rectangle's dimensions are doubled, its area is multiplied by 4.

d.

10	6	60
↓ · 2	↓ · 2	↓ · 4
20	12	240

← $60 \cdot 4 \overset{?}{=} 240$ ✔

The area of the new rectangle is 4 times the area of the original rectangle. The conjecture works.

EXERCISES

1. a. Find the area of the triangle at the right.
b. Find the area of a triangle with dimensions $\frac{1}{2}$ of those at the right.
c. Make a conjecture about how the area changes as the dimensions are halved, and then test your conjecture with a new triangle.

6 cm   8 cm
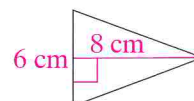
9-3 Patterns and Tables

What You'll Learn

OBJECTIVE 1 To represent patterns in tables

OBJECTIVE 2 To write rules using tables

...And Why

To find gasoline mileage, as in Example 1

✔ Check Skills You'll Need

❓ For help, go to Lessons 2-3 and 2-4.

Solve each equation.

1. $x + 4 = -6$
2. $7t = 112$
3. $5y = -135$
4. $a + (-3) = 17$
5. $36 + p = 118$
6. $-18r = -18$

7. **Reasoning** Complete the equation with one operation followed by one number.
   $2 \;\blacksquare\; \blacksquare = 12$

OBJECTIVE 1

ⓘTEXT Interactive lesson includes instant self-check, tutorials, and activities.

Representing Patterns in Tables

You can use a table to represent a number pattern.

1 EXAMPLE **Representing a Pattern** 🌐 **Real World**

Gas Mileage The table below shows the distance a new car can go using different amounts of gasoline. How far can the car go using 4 gal of gasoline? How far can it go using 5 gal?

Gas (gal)	Miles Driven
1	18.1
2	36.2
3	54.3
4	
5	

= 1 × 18.1
= 2 × 18.1
= 3 × 18.1

The values in the second column are 18.1 times the values in the first column.

$4(18.1) = 72.4$ and $5(18.1) = 90.5$

The car can go 72.4 mi using 4 gal and 90.5 mi using 5 gal.

Real-World 🌐 Connection

Hybrid cars improve fuel efficiency by using both gasoline and electricity.

✔ Check Understanding

1 a. Copy and complete the table.
   b. Find the cost of 20 lb of fresh fish.
   c. **Reasoning** The relationship between values is the same for each row of the table in Example 1. Explain why this helps you find the pattern.

Fresh Fish (lb)	Price ($)
1	6.50
2	13.00
3	19.50
4	
5	

You can use the relationship between values in the rows of a table to find missing values.

2 EXAMPLE **Finding the Values of Variables**

Find the values of p and q in the table at the right.

What are the possible relationships between the quantities in a row?

First row: $1 + 17 = 18$ or $1 \cdot 18 = 18$
Second row: $4 + 68 = 72$ or $4 \cdot 18 = 72$

The rule that works for both rows is *Multiply by 18*.

$7 \cdot 18 = p$ $q \cdot 18 = 270$ ← **Find p and q using the rule.**
$126 = p$ $q = 15$

Check You can check your rule by testing another row of values.

$12 \cdot 18 = 216$ ✔   ← **The answer checks.**

A	B
1	18
4	72
7	p
12	216
q	270

✔ **Check Understanding** ② Find the values of the variables in the table at the right.

C	3	5	x	9	11
D	66	110	132	198	y

OBJECTIVE

2

Writing Rules Using Tables

A table can help you write a variable expression that describes a sequence.

3 EXAMPLE **Using a Table With a Sequence**

Write an expression to describe the rule for the sequence $8, 16, 24, 32, \ldots$. Then find the 100th term in the sequence.

8	16	24	32	. . .
↑	↑	↑	↑	
Term 1	Term 2	Term 3	Term 4	and so on

Make a table that pairs the term numbers and the values in the sequence.

Term Number	1	2	3	4	· · ·	n
	↓·8	↓·8	↓·8	↓·8	↓·8	↓·8
Value of Sequence	8	16	24	32	· · ·	◼

In words, the rule is *Multiply the term number by 8*.
If you let $n =$ the term number, you can write this rule as $n \cdot 8$, or $8n$.

$n \cdot 8$ ← **Write an expression for the rule.**

$100 \cdot 8 = 800$ ← **Use n = 100 to find the 100th term.**

✔ **Check Understanding** ③ **a.** Write an expression for the rule for $-8, -7, -6, -5, \ldots$. Find the 100th term in the sequence.
b. **Mental Math** The rule for a sequence is $100n$, where n is the term number. Find the 100th term.

A Practice by Example

Example 1 (page 480)

Copy and complete each table.

1.
Cans of Soup	Number of Servings
3	9
4	12
5	15
6	▪
7	▪

2.
Beads (dozen)	Cost ($)
1	0.48
2	0.96
3	1.44
4	▪
5	▪

3.
Change in a Parking Meter ($)	Time Allowed to Park (h)
0.25	0.5
0.50	1
0.75	▪
1.25	▪

4.
Miles	Time (h)
10	0.4
20	0.8
30	1.2
40	▪
50	▪

Example 2 (page 481)

Find the values of the variables in each table.

5.
A	B
7	84
9	108
11	132
15	m
n	240

6.
C	D
25	−5
50	−10
75	−15
p	−18
120	q

7.
E	F
32	40
96	120
112	s
144	180
t	230

Example 3 (page 481)

Write a variable expression to describe the rule for each sequence. Then find the 100th term.

8. 11, 22, 33, 44, . . .

9. −19, −18, −17, −16, . . .

10. $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, . . .

11. −3, −6, −9, −12, . . .

12. −18, −36, −54, −72, . . .

13. 100, 200, 300, 400, . . .

B Apply Your Skills

🌐 14. **Gasoline** One month's average price for regular unleaded gasoline is $1.20 per gallon. Using this relationship, make a table that shows the price for 5, 10, 15, and 25 gallons of regular unleaded gasoline.

🌐 15. **Music** The table shows costs for violin lessons. Copy and complete the table.

Time (h)	0.5	1	1.5	2
Cost ($)	12.50	▪	▪	▪

16. **Measurement** There are 36 inches in 1 yard. Make a table that shows the number of yards in 72 inches, 180 inches, and 288 inches.

17. **a.** Copy and complete the table.
b. **Writing in Math** Write a rule in words to find y when you know x.

x	0	1	2	3	4	▪
y	−1	2	5	8	▪	14

18. Temperature The relationship between Kelvin (K) and Celsius (C) temperatures is $K = 273 + C$. Make a table of Kelvin temperatures for Celsius temperatures of 0°, 10°, 20°, 40°, 80°, and 120°.

Find the values of the variables in each table.

19.

A	0	10	100	1,000	y
B	0	0.05	x	5	50

20.

C	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	n	$\frac{1}{6}$
D	$\frac{1}{3}$	m	$\frac{1}{6}$	$\frac{2}{15}$	$\frac{1}{9}$

21. **Geometry** Use the relationship between side length and area to make a table for the areas of squares with sides of 2, 3, 5, 8, 10, and 12 in.

22. **a. Patterns** Make a table that shows the number of blue and yellow squares for each group.
   b. How many blue squares will be in group 10?

23. Use the relationship $y = \frac{3}{2}x$.
   a. Make a table that shows the values of y for $x = -2, -4, 2,$ and 4.
   b. Find the value of x for $y = 0$.

24. One $cm^3 = 1$ mL. Use this relationship to make a table showing the number of liters in 3 cm^3, 300 cm^3, and 3,000 cm^3.

25. **Stretch Your Thinking** Using only addition and subtraction and just three operation signs, write an expression that equals 100 using the digits shown. Do not change the order of the digits.   1 2 3 4 5 6 7 8 9

Real-World Connection

The freezing point of water is 0°C, or 273 K.

C Challenge

Test Prep

Gridded Response

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Online lesson quiz at
www.PHSchool.com
Web Code aba-0903

Use the table. Find the value of y for each value of x.

x	y
0	3
2	5
4	7

26. -2 27. 0.2

28. 5   29. 7

30. 20   31. 100

Mixed Review

Lesson 4-4   **Find each product.**

32. $\frac{1}{4}$ of 28   33. $\frac{3}{5} \cdot 25$ 34. $\frac{2}{7} \cdot \frac{21}{50}$ 35. $4\frac{2}{3} \cdot 4\frac{1}{2}$

Lesson 1-8   **Find each product.**

36. $-4 \cdot (-4)$ 37. $20 \cdot (-9)$ 38. $250 \cdot (-5)$ 39. $-11 \cdot (-11)$

9-4 Function Rules

What You'll Learn

OBJECTIVE 1 To write function rules

OBJECTIVE 2 To evaluate functions

. . . And Why

To find a rule for driving distance, as in Example 1

OBJECTIVE

1

Writing Function Rules

TEXT Interactive lesson includes instant self-check, tutorials, and activities.

Suppose you are riding in a car at an average speed of 50 mi/h. You can expect to travel 100 mi in 2 h, since $2 \cdot 50 = 100$.

Input (time)

The distance you travel depends on the driving time. The "function machine" at the right shows the relationship between time (input) and distance (output).

A **function** is a relationship that assigns exactly one output value for each input value.

Output (distance)
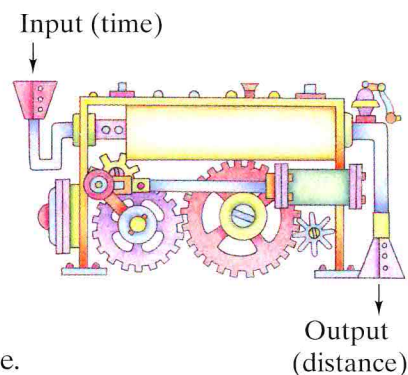
1 EXAMPLE   Real-World 🌐 Problem Solving

Travel You are traveling in a car at an average speed of 55 mi/h. Write a function rule for the relationship between the time and distance you travel.

You can **make a table** to solve this problem.

Input: time (h)	1	2	3	4
Output: distance (mi)	55	110	165	220

Reading Math

An output is a function of the input.

distance in miles $= 55 \cdot$ time in hours   ← **Write the function rule in words.**

$\qquad\qquad\qquad d = 55t$ ← **Use variables d for distance and t for time.**

✔ **Check Understanding** **1** **a.** Write a function rule for the relationship between time and distance for an average speed of 62 mi/h.

b. **Reasoning** What type of sequence do the outputs in Example 1 form?

The variables x and y are often used to represent input and output. You can describe the relationship between the values in the table in three ways.

Input	Output
1	4
2	5
3	6
4	7

Each output is 3 greater than the input.

Output = Input + 3

$$y = x + 3$$

You can write a function rule by looking for patterns in a function table.

② EXAMPLE **Writing Function Rules**

Write a rule for the function represented by each table.

a.

x	y
0	0
1	−4
2	−8
3	−12

When $x = 0$, $y = 0$.
Each y equals
−4 times x.

b.

x	y
0	−3
1	−1
2	1
3	3

When $x = 0$, $y = −3$.
Each y equals 2
times x, plus −3.

The function rule is
$y = −4x + 0$, or $y = −4x$.

The function rule is
$y = 2x + (−3)$, or $y = 2x − 3$.

✔ **Check Understanding ②** Write a rule for the function represented by the table at the right.

x	0	1	2	3
y	1	5	9	13

OBJECTIVE

2 **Evaluating Functions**

Given a function rule, you can evaluate the function for any input value.

③ EXAMPLE **Finding Input/Output Pairs**

Use the function $y = −3x + 5$. Find y for $x = 0, 1, 2,$ and 3. Then make a table for the function.

$y = −3(0) + 5 = 5$ ← Substitute 0, 1, 2, and 3 for x.
$y = −3(1) + 5 = 2$
$y = −3(2) + 5 = −1$ List the values in a table. →
$y = −3(3) + 5 = −4$

x	y = 3x + 5
0	5
1	2
2	−1
3	−4

✔ **Check Understanding ③** Use the function rule $y = 2x − 4$. Find y for $x = 0, 1, 2,$ and 3. Then make a table for the function.

A Practice by Example

Example 1
(page 484)

Write a function rule that represents each relationship.

1. the time t and the distance d you travel at an average speed of 30 mi/h

2. the number n of words you type and the time t it takes, at a typing rate of 32 words/min

3. the amount c of energy you burn and the time t you spend exercising, burning Calories at a rate of 12 Cal/min

Example 2
(page 485)

Write a rule for the function represented by each table.

4.

x	y
0	0
1	5
2	10
3	15

5.

x	y
0	4
1	5
2	6
3	7

6.

x	y
0	5
1	8
2	11
3	14

7.

x	y
0	1
1	−8
2	−17
3	−26

8.

x	y
0	−2
1	0
2	2
3	4

9.

x	y
0	0
1	−8
2	−16
3	−24

Example 3
(page 485)

Use each function rule. Find y for $x = 0, 1, 2,$ and 3. Then make a table for the function.

10. $y = x + 2$

11. $y = 12 - 2x$

12. $y = 4x$

13. $y = x \div 2$

14. $y = 9 - x$

15. $y = -3x$

16. $y = 2x + 1$

17. $y = 4x - 2$

18. $y = x^2 + 1$

B Apply Your Skills

19. **Writing in Math** Choose one of the tables from Exercises 4–9. Explain what patterns you noticed and how you found the rule.

20. a. Measurement Write a rule for the number of inches n in f feet.
b. Use your function rule to find the number of inches in 7 ft.

Explain whether each situation represents a function.

21. Input: the number of pounds a turkey weighs
Output: the number of $\frac{1}{4}$-lb servings

22. Input: the number of days in a month
Output: the number of Tuesdays in the month

? Need Help?
Remember that a function has only one output for each input.

23. Reading A student can read 150 words in one minute.
  a. Write a function rule to represent the relationship between the number of words and the time in which they are read.
  b. How many words can the student read in 8 minutes?
  c. How long would it take the student to read 2,850 words?
  d. Reasoning Is your answer to part (c) an input or an output? Why?

Use the function rule $y = \frac{2}{x}$. Evaluate the function for each value of x.

24. -2 **25.** -1 **26.** 10   **27.** -3

28. Money Suppose you put $.50 in your piggy bank on July 1, $1.00 on July 2, $1.50 on July 3, and so on. Use n to represent the date. Write a function rule for the amount you put in for any date in July.

Write a rule for the function represented by each table.

29.

x	y
0	0
1	$\frac{1}{2}$
2	1
3	$1\frac{1}{2}$

30.

x	y
0	$-\frac{1}{2}$
1	$1\frac{1}{2}$
2	$3\frac{1}{2}$
3	$5\frac{1}{2}$

31.

x	y
0	1
1	$1\frac{1}{2}$
2	2
3	$2\frac{1}{2}$

32.

Laundry Loads	Cost ($)
1	2.75
2	5.50
3	8.25
4	11.00

33.

Time (h)	Kangaroo's Distance (km)
2	96
4	192
6	288

C Challenge

Use the function rule $y = x^2 + 2x$. Evaluate the function for each value of x.

34. -0.25 **35.** $-\frac{1}{2}$ **36.** $1\frac{1}{4}$ **37.** -0.2

38. Use the function machine.
  a. Make an input/output table for integer inputs from -5 to 5.
  b. Which two input values result in an output of 22?
  c. Reasoning If two input values produce the same output, is the rule a function? Explain.

Input

$x^2 + x + 2$

Output
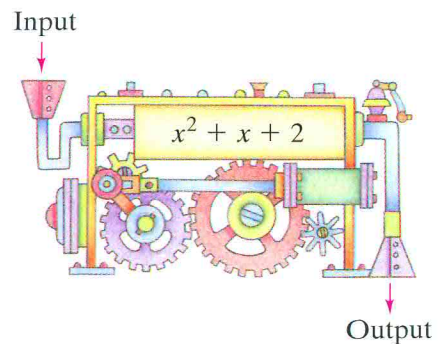
39. Stretch Your Thinking When 4 is subtracted from a certain mixed number less than 10, the result is the same as the quotient of that mixed number and 4. What is the mixed number?

Multiple Choice

40. Which function rule describes the number of feet f as a function of the number of yards y?

   A. $f = \frac{1}{3}y$ **B.** $f = 3y$ **C.** $f = 6y$ **D.** $f = 12y$

41. Use the function rule $y = 6x - 1$. Find the value of y for $x = 1$.

   F. -7 **G.** -5 **H.** 5   **I.** 7

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Web Code aba-0904

42. Which rule represents the function in the table below?

   A. $s = 6.5h$ **B.** $h = 6.5 + s$ **C.** $h = 13s$ **D.** $s = 13h$

Hours Worked	1	2	3	4
Salary ($)	6.50	13.00	19.50	26.00

Short Response

43. a. Write a rule for the function that represents the total cost of an item with a 5% sales tax.

   b. Use your rule to find the total cost of a $15 item.

Mixed Review

Lesson 6-7

Find each percent of decrease. Round to the nearest tenth of a percent.

44. 100 to 80   **45.** 7 to 3   **46.** 400 to 250

Lesson 6-4

Find each answer.

47. 8% of 12   **48.** 70% of 150   **49.** 120% of 60

Checkpoint Quiz 1   Lessons 9-1 through 9-4

TEXT Instant self-check quiz online and on CD-ROM

Write a rule for each sequence. Then find the next three terms.

1. $7, 14, 21, 28, \ldots$ **2.** $250, 220, 190, 160, \ldots$

3. $2, 5, 11, 23, \ldots$ **4.** $-4, 12, -36, 108, \ldots$

5. Identify each sequence in Exercises 1–4 as *arithmetic*, *geometric*, *both*, or *neither*.

For Exercises 6–10, use the table at the right.

6. Find the values of m and n.

7. Graph the values in the table.

8. Use the graph to estimate the value of x for $y = 10$.

9. Write a rule for the function.

10. Find the value of y for $x = 11$.

x	y
2	7
4	13
5	m
7	22
n	28

9-5

Using Tables, Rules, and Graphs

What You'll Learn

OBJECTIVE 1 To graph functions

...And Why

To find plant growth, as in Example 2

✔ Check Skills You'll Need

? For help, go to Lesson 9-4.

Make a table of values for each function. Use inputs of 0, 1, 2, and 3.

1. $y = 2x$

2. $y = -x$

3. $y = 4x - 2$

4. $y = x + 5$

5. $y = \frac{1}{2}x$

6. $y = -3x$

OBJECTIVE

1

Graphing Functions

i TEXT Interactive lesson includes instant self-check, tutorials, and activities.

Investigation: Graphing a Function

1. Count the number of times you breathe in one minute. Repeat four times and record your results. Find the mean of your results.

2. Copy and complete the table, using your mean.

t (min)	1	2	3	4	5	6
Number of Breaths						

3. Write a function rule.

4. Draw a graph of your data. Put the number of minutes on the horizontal scale and the number of breaths on the vertical scale.

The graph of a function shows the relationship between inputs and outputs.

1 EXAMPLE Graphing Using a Function Table

Graph the function represented by the table at the left.

Input x	Output y
0	3
1	5
2	7
3	9
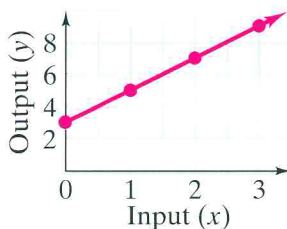
Graph input x on the horizontal axis and output y on the vertical axis.

Draw a line through the points.

✔ Check Understanding 1 Graph the function represented by the table at the right.

Input x	0	1	2	3
Output y	5	8	11	14

You can also graph a function using its function rule.

2 EXAMPLE **Real-World** **Problem Solving**

Plants A plant grows 1.38 cm for each hour of sunlight it receives. Write and evaluate a function rule to find the growth of the plant when it receives 15 h of sunlight. Then graph the function.

Step 1 Write a function rule.

Words	Growth	equals	1.38	times	hours of sunlight

Let g = growth in centimeters. Let s = hours of sunlight.

Equation	g	$=$	1.38	$\cdot$	s

Step 2 Evaluate the function.

$g = 1.38 \cdot 15 = 20.7$ ← **Substitute 15 for s.**

The plant grows 20.7 cm when it receives 15 h of sunlight.

Step 3 Make a table of values and graph the function.

Hours of Sunlight (s)	Centimeters of Growth (g)
1	1.38
2	2.76
3	4.14
4	5.52
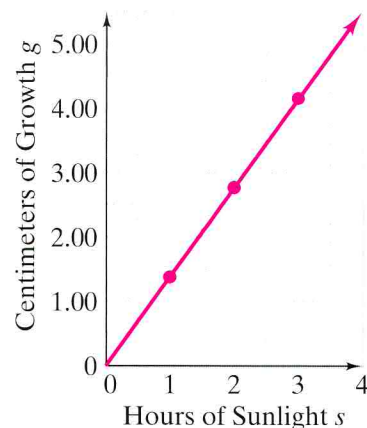
✔ **Check Understanding** **2** **a.** The cost of a long distance telephone call is a function of time in minutes. A state-to-state call costs 13¢/min. Write and evaluate a function to find the cost of a 15-min call. Then graph the function.

b. **Reasoning** Does every function have an output of 0 for an input value of 0? Explain.

EXERCISES

A Practice by Example

Example 1
(page 489)

Graph the function represented by each table.

1.

Input x	Output y
0	1
1	4
2	7
3	10

2.

Input x	Output y
0	0
1	12
2	24
3	36

Write and evaluate a function to find the output for an input of 10 h. Then graph the function.

🌐 **3. Employment** Total salary S is a function of the number t of hours worked. Your salary is $6/h.

🌐 **4. Calories** Total Calories c burned by walking is a function of the number t of hours spent walking. You burn an average of 300 Cal/h.

🌐 **5. Air Travel** Total distance d is a function of the number t of hours traveled. Airplane speed averages 320 mi/h.

Ⓑ Apply Your Skills

Graph each function. Use input values of 1, 2, 3, 4, and 5.

6. $y = 5x$ **7.** $y = 2x + 1$ **8.** $y = x \div 2$

9. Open-Ended Choose one function rule from Exercises 6–8. Describe a situation that could represent the function.

🌐 **10. Flight** Amelia Earhart set several flight speed records. The table models the relationship between distance and time for a flight at Earhart's record speed.
    a. Write a rule for the function represented by the table.
    b. Find the average speed. Justify your answer.
    c. Estimate the number of hours it would take to fly 1,890 mi.
    d. Graph the function.

Amelia Earhart's Flight

Time (h)	Distance (mi)
2	362
4	724
6	1,086
8	1,448

Real-World 🌐 Connection

In 1928, Amelia Earhart became the first woman to fly across the Atlantic Ocean.

Match each graph with a function rule.

11.

12.

13.
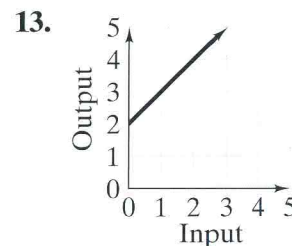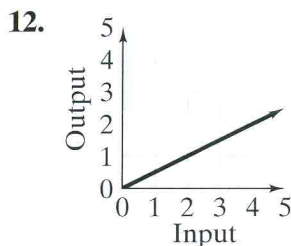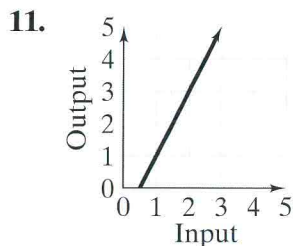
A. Output = $\frac{1}{2} \cdot$ Input

B. Output = Input + 2

C. Output = $2 \cdot$ Input $- 1$

14. Writing in Math For each situation, which would best represent a function—a table, a rule, or a graph? Explain your choice.
    a. You only have a few values or you do not know the rule.
    b. You have many input and output values.
    c. You want to see the relationship between the values.

15. a. Data File, p. 467 It costs $380 per year to feed a cat. Write a function rule for the cost c of feeding a cat during its lifetime y.
    b. Evaluate the function to find the cost of feeding a cat that lives to the maximum life span.

16. a. Make a function table for the graph.
b. Write a rule for the function.
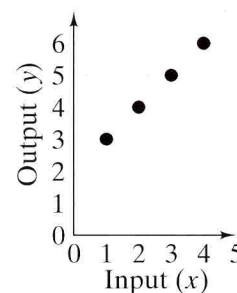
Evaluate $y = 2x^2 - 8$ for each value of x.

17. 3   **18.** 0   **19.** -2

20. -3 **21.** 2   **22.** $\frac{1}{2}$

C **Challenge**

23. Geometry The area of a square is a function of the length of a side. Write a rule and graph the function. Describe the shape of your graph.

24. Stretch Your Thinking You have a total of 12 cards in 4 different colors. The probability of drawing a green card is the same as the probability of drawing a yellow card. The probability of drawing a red card is $\frac{1}{4}$ and of drawing a blue card is $\frac{1}{12}$. How many green cards are in the box?

Test Prep

Multiple Choice

25. Which number can be an output for the function $y = 2x - 1$ if only whole numbers are used as inputs?
A. 6   **B.** 8   **C.** 11   **D.** 14

26. Which output value would NOT be on the graph of the function $y = -4x + 20$ for input values of 0, 1, 2, 3?
F. 0   **G.** 8   **H.** 12   **I.** 16

27. Which situation CANNOT be modeled by $y = 4x$?
A. the cost of a call at 4¢/min
B. the perimeter of a trapezoid
C. your pay after working 4 h
D. the number of stamps in 4 packs

Extended Response

28. You ride a bicycle at 12 mi/h. The distance you ride is a function of time.
a. Copy and complete the table.
b. Graph the function.
c. How many miles do you ride in 3.5 h?

Time (h)	Distance (mi)
1	12
1.5	18
2	a
2.5	b

Mixed Review

Lesson 3-6   **Compare. Use $<$, $>$, or $=$.**

29. $\frac{7}{9} \; \blacksquare \; \frac{3}{4}$ **30.** $\frac{5}{14} \; \blacksquare \; \frac{2}{6}$ **31.** $\frac{7}{30} \; \blacksquare \; \frac{1}{3}$ **32.** $\frac{8}{24} \; \blacksquare \; \frac{2}{6}$

Lesson 1-3   **Find the value of each expression.**

33. $(95.26)(110)$ **34.** $(0.23)(8.45)$ **35.** $0.384 \div 9.6$ **36.** $19.563 \div 6$

Three Views of a Function

When you input a function rule in a graphing calculator, you can view the graph of the function or a table of values.

EXAMPLE

Graph $y = 9 - x$ and make a table of values.

Step 1 Use WINDOW to set the range.

```
WINDOW FORMAT
Xmin = 0
Xmax = 10
Xscl = 1
Ymin = 0
Ymax = 10
Yscl = 1
```

Step 2 Use Y= to enter the function.

```
Y1 = 9-X
Y2 =
Y3 =
Y4 =
```

Step 3 Use the GRAPH feature to view the graph.
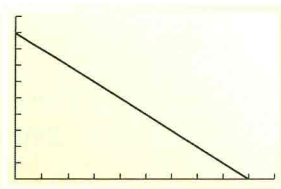
Step 4 Use the TABLE feature to make a table of values.

X	Y1
0	9
1	8
2	7
3	6
4	5
5	4
6	3

X=0

Step 5 Sketch the graph and copy the table of values.

x	y
0	9
1	8
2	7
3	6
4	5
5	4
6	3
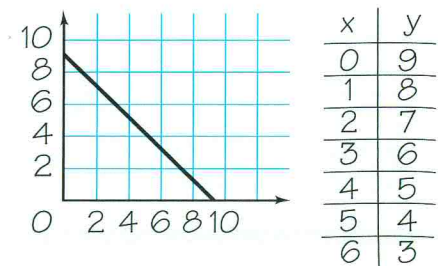
EXERCISES

Use a graphing calculator to graph each function and make a table of values. Sketch the graph and copy the table of values.

1. $y = 2x$

2. $y = x - 3$

3. $y = 13 - 2x$

4. $y = x + 1$

5. $y = 3x - 4$

6. $y = 0.5x + 6$

7. $y = 7$

8. $y = x^2$

9. $y = x^2 - 8x + 16$

9-6

Interpreting Graphs

What You'll Learn

OBJECTIVE 1 To describe a graph

OBJECTIVE 2 To sketch a graph from a description

. . . And Why

To graph a trip to school and back, as in Example 2

✔ **Check Skills You'll Need**

🌵 For help, go to Lesson 9-4.

Find the distance for the following times. Use $d = rt$ with $r = 40$ mi/h.

1. 2 h   **2.** 2.5 h   **3.** 5 h

4. 3.5 h   **5.** $\frac{1}{4}$ h   **6.** $\frac{1}{2}$ h

7. Reasoning What does the graph of the function $d = 40t$ look like?

OBJECTIVE

1 **Describing Graphs**

📱**TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

When you graph a function rule, you can see how one quantity changes compared to another.

1 EXAMPLE **Describing a Graph** 🌐 **Real World**
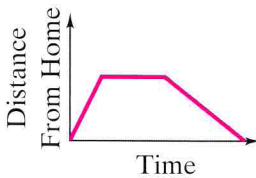
Shopping You ride your bike to the supermarket and shop. The graph at the left relates time and your distance from home. Describe what you can tell about the trip from the steepness of the lines.

Biking quickly to the supermarket, distance increases quickly

Shopping

Biking slowly home with groceries
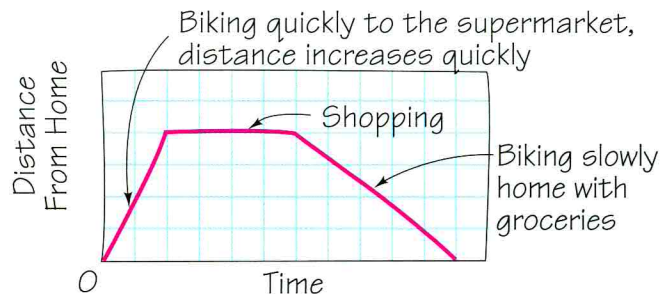
A steeper line on the graph shows faster speed. A horizontal line represents a period of no change in distance from home.

✔ **Check Understanding** **1** **a.** You live 6 blocks from school. The graph at the right shows your walk home on a sunny day. Describe what the graph shows.

b. **Reasoning** During which period(s) of time was your speed the greatest? Explain.
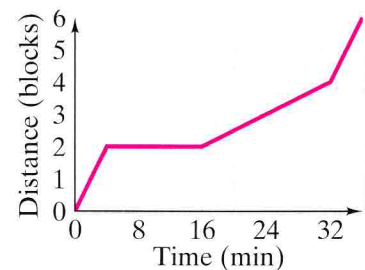
You can sketch a graph to describe a real-world situation.

2 EXAMPLE   **Sketching a Graph**   🌎 **Real World**

Transportation Ciara's mom drove her part of the way to school. Ciara waited for a friend and walked the rest of the way to school. She took a bus home. Sketch a graph to show the distance Ciara traveled compared to time.
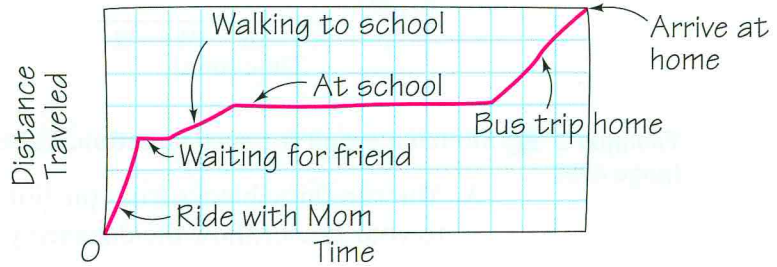
✔ **Check Understanding** **2 a.** Sketch a graph of the situation in Example 2 using *Distance from Home* instead of *Distance Traveled* for the vertical axis.
b. Reasoning How is your graph different from the one in Example 2?

When you draw a graph, you may need to consider what is reasonable in a real-world context.

3 EXAMPLE   **Real-World** 🌎 **Problem Solving**

Games The cost to play games at an arcade is given in the table. Make a graph of the cost to play 1, 2, 3, and 4 games at the arcade.

Step 1 Make a table.

Number of Games	Cost ($)
1	2.00
2	3.50
3	5.00
4	6.50

Step 2 Graph. Do not connect the points.
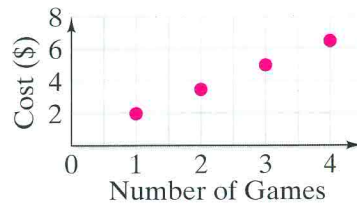
Check for Reasonableness Each cost is for playing an entire game. You cannot pay for a part of a game. It is reasonable not to connect the points.

✔ **Check Understanding** **3 a.** The table shows the number of cans in the cafeteria juice machine over time. Graph the data.
b. Does it make sense to connect the points in this graph? Explain.

Time	Number of Cans
8 A.M.	30
9 A.M.	20
10 A.M.	19
11 A.M.	19

A **Practice by Example**

Describe what each graph shows.

Example 1
(page 494)

1. Walking Home From School

Distance (blocks) vs Time (min)

2. Distance From Home

Distance (mi) vs Time (h)
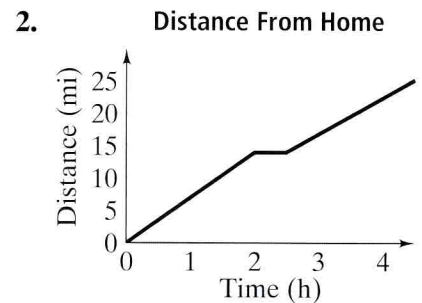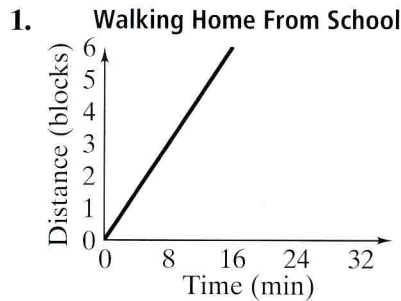
Example 2
(page 495)

🌎 **Sketch a graph for each situation. Label each section and each axis.**

3. You run three blocks from the library and then walk five more blocks to your home. Show the distance you travel on the vertical axis.

4. You ride your bike slowly up a steep hill, and then quickly down the other side. Show your speed on the vertical axis.

5. You climb a jungle gym and then slide down the slide. Show the distance from the ground on the vertical axis.

6. You cycle 2 mi, stop at a traffic light, and then proceed at your original speed for another mile. Show the distance you travel on the vertical axis.

Example 3
(page 495)

Graph the data. Should you connect the points on each graph? Explain.

7. Lemonade Sales

Cups Sold	Income ($)
1	0.75
2	1.50
3	2.25
4	3.00
5	3.75

8. Miles From Home

Time (h)	Miles
1	60
2	85
3	120
4	180

B **Apply Your Skills**

🌎 **9. Library** You pay a 5-cent fine for each day a library book is overdue.
   a. Graph the cost of fines for a book overdue by 1, 2, 3, 4, and 5 days.
   b. Did you connect the points on your graph? Explain.

10. Suppose you are steadily pouring sand into the bowl at the left. Which graph below better shows the relationship of the height of the sand with the amount you have poured? Why?
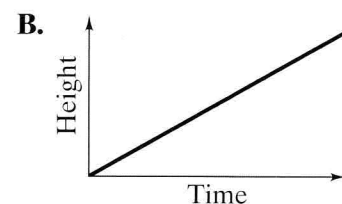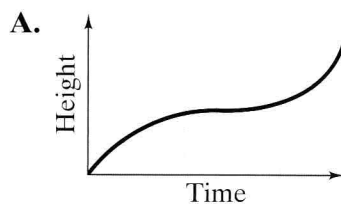
A.

Height vs Time

B.

Height vs Time

Match each graph with the situation that describes it.

11.

Time

12.

Time

13.

Time
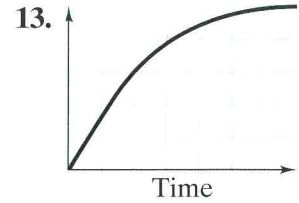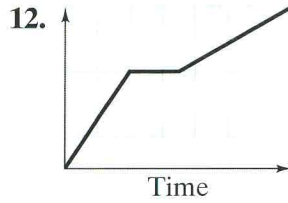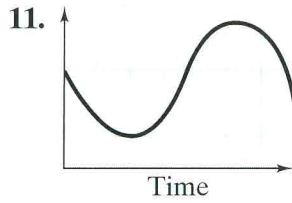
A. height of a person from birth to age 20

B. air temperature in a 24-hour period starting at midnight

C. distance raced with a fall over a hurdle

14. Sports The graph shows a 90-m race. One student starts 5 s after the other.
   a. Describe what the graph shows.
   b. Who wins the race?
   c. **Reasoning** If the lines were parallel, what would the graph tell you about who wins the race?
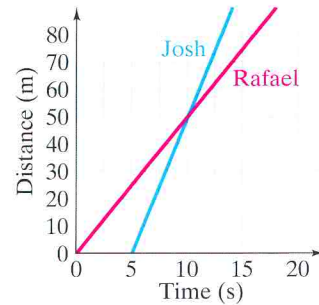
Sketch a graph for each situation.

15. A flag is raised on a flagpole.

16. Judy takes a walk to the mall. She stops to visit a friend. Judy continues walking to the mall, shops for a while, and then runs home for dinner.

17. Writing in Math Lee, Paulo, and Mary each walk their dogs six times around the block. Who walks the fastest? Justify your answer.
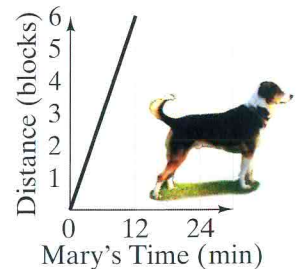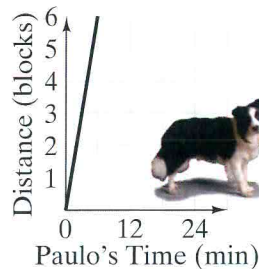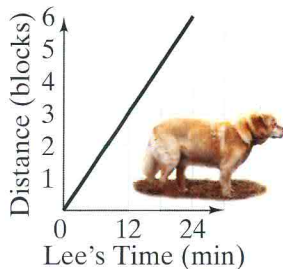
Sketch a graph for each situation. Label the axes.

18. Geometry As a square's dimensions increase, its perimeter increases.

19. Nutrition The more crackers you eat, the more Calories you consume.

20. Reasoning A driver sets a car's cruise control to 50 mi/h. Which graph shows the car's speed? Which shows the distance traveled? Explain.

A.

Time

B.
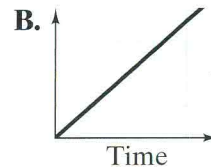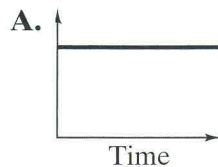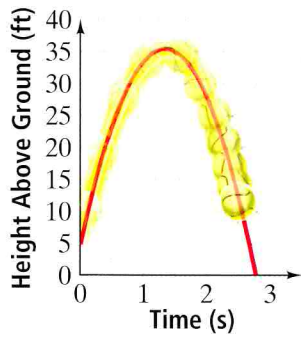
Time

Estimation Refer to the graph at the left. It shows what happens when a ball is thrown in the air.

Height Above Ground (ft) — 40, 35, 30, 25, 20, 15, 10, 5, 0 — **Time (s)** 0 1 2 3
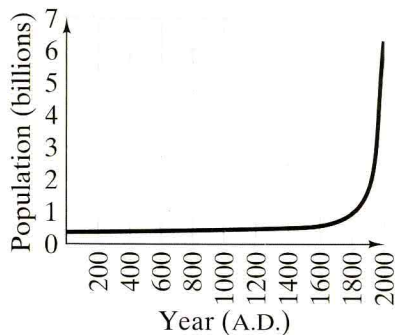
21. When does the ball hit the ground?

22. Why are there two times when the ball's height is 20 ft? What are they?

23. When the time is 0, the height of the ball is *not* 0. Why?

24. **Stretch Your Thinking** The last digit of a four-digit number is twice the first. The second is three less than the third. The third is the sum of the first and last. Each digit is different and none is 0. What number is it?

Test Prep

Reading Comprehension   Read the passage and answer the questions below.

Population (billions) — 7, 6, 5, 4, 3, 2, 1, 0 — **Year (A.D.)** 200 400 600 800 1000 1200 1400 1600 1800 2000

The Population Explosion

From A.D. 1 to 1650, the world population grew by 300 million. The average growth was just 180,000 people per year, or about 21 people per hour. Today twelve babies are born every three seconds, but only five people die. This means that the population is growing by about 8,400 people per hour, or about 73.6 million per year.

25. Which is a function rule for converting population growth from per hour to per year?
   A. $y = 60 \cdot 24 \cdot 12 \cdot x$ **B.** $y = 24 \cdot 12 \cdot 12 \cdot x$
   C. $y = 52 \cdot 24 \cdot x$ **D.** $y = 24 \cdot 365 \cdot x$

Take It to the NET
Online lesson quiz at
www.PHSchool.com
Web Code: aba-0906

26. What is the approximate daily population growth today?
   F. 6.1 million   **G.** 3.1 million   **H.** 201,600   **I.** 8,400

27. Explain how the graph illustrates the data in the article.

Mixed Review

Lesson 8-2   **Find the area of a triangle with the given base and height.**

28. 5 m, 2 m   **29.** 10 ft, 3 ft   **30.** 18 cm, 6 cm   **31.** 19 in., 6 in.

Lesson 6-5   **Algebra**   **Solve.**

32. What is 2% of 12?   **33.** What percent of 150 is 50?

34. What percent of 60 is 12?   **35.** What is 120% of 25?

9-7 Simple and Compound Interest

What You'll Learn

OBJECTIVE 1 To find simple interest

OBJECTIVE 2 To find compound interest

...And Why

To find the balance in an account, as in Example 3

Change each percent to a decimal.

1. 4%   **2.** 9%   **3.** 2.0%

4. 6.5%   **5.** 5.09%   **6.** 18.21%

New Vocabulary • principal • simple interest • compound interest • balance

---

OBJECTIVE

1 Finding Simple Interest

iTEXT Interactive lesson includes instant self-check, tutorials, and activities.

Money does not grow on trees, but it does grow in a bank. When you deposit money, you earn money, called "interest." When you borrow, you pay interest.

The original amount you deposit or borrow is the **principal.** Interest calculated only on the principal is **simple interest**.

You can use a formula to calculate simple interest.

> **Key Concepts** — **Simple Interest Formula**
>
> $$I = prt$$
>
> I is interest, p is principal, r is annual rate of interest, and t is time in years.

1 EXAMPLE **Finding Simple Interest**

Find the simple interest on $300 borrowed for 3 years at an annual interest rate of 4%.

$I = prt$ ← Write the formula.

$I = (300)(0.04)(3)$ ← Substitute. Use 0.04 for 4%.

$\quad = 36$ ← Simplify.

The interest is $36.

Check Understanding 1 **a.** Find the simple interest on a $220 loan at a 5% annual rate for 4 years.
b. **Reasoning** Of what quantities is interest a function?

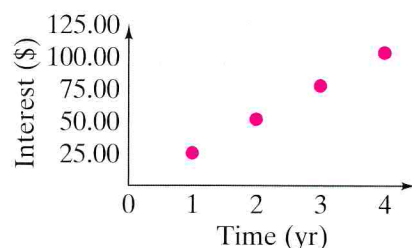A graph of simple interest shows the increase in interest earned over time.

2 EXAMPLE Graphing Simple Interest

You have $500 in an account at an annual rate of 5.1%. Interest is paid and withdrawn at the end of each year. Graph the total interest you earn after 1, 2, 3, and 4 years.

Step 1 Make a table. Let t = time.   **Step 2** Draw a graph.

Time (yr)	Interest ($)
1	25.50
2	51.00
3	76.50
4	102.00

✔ **Check Understanding** ② Graph the simple interest you earn on $950 at an annual rate of 4.2%.

OBJECTIVE

2 Finding Compound Interest

Compound interest is interest that is paid on the original principal and on any interest that has been left in the account. The **balance** of an account is the principal plus the interest earned.

> **Key Concepts**   **Compound Interest Formula**
>
> $$B = p(1 + r)^t$$
>
> B is balance, p is principal, r is annual interest rate, and t is time in years.

3 EXAMPLE Real-World 🌐 Problem Solving

Banking Suppose you deposit $5,000 in a bank account that pays 3.75% interest compounded annually. What is your balance after 9 years?

$$B = p(1 + r)^t \qquad \leftarrow \text{Write the formula.}$$
$$= 5,000(1 + 0.0375)^9 \qquad \leftarrow \text{Substitute. Use 0.0375 for 3.75\%.}$$
$$\approx 5,000(1.392813439) \qquad \leftarrow \text{Use a calculator to simplify the power.}$$
$$= 6,964.07 \qquad \leftarrow \text{Round to the nearest cent.}$$

The balance after 9 years is $6,964.07.

Need Help?
For help using the Order of Operations with exponents, go to Lesson 3-1.

Check for Reasonableness The compound interest is $6,964.07 − $5,000, or about $1,964. Simple interest would be $I = (\$5,000)(0.0375)(9)$, or about $1,688. Since compound interest is greater, the answer is reasonable.

✔ **Check Understanding** ③ You deposit $3,000 in a bank account that pays 4.25% interest compounded annually. What is your balance after 12 years?

A **Practice by Example**

Example 1
(page 499)

Find the simple interest on a $340 loan at each rate.

1. 7% annual interest, 3 years   **2.** 12% annual interest, 5 years

3. 15% annual interest, 1 year   **4.** 6% annual interest, 6 years

Example 2
(page 500)

Graph the total simple interest earned for each amount over 4 years.

5. $500 at 4.5%   **6.** $1,200 at 6.5%   **7.** $375 at 5.75%

8. $200 at 5.0%   **9.** $2,000 at 10%   **10.** $2,000 at 0.5%

Example 3
(page 500)

Find the balance in each compound interest account.

11. $1,400 after 3 years at 5.5%   **12.** $1,800 after 11 years at 6.0%

13. $900 after 10 years at 4.62%   **14.** $2,500 after 50 years at 2.2%

B **Apply Your Skills**

15. Loans You borrow $500 at 18% annual compound interest. You make no payments for 6 months. How much do you owe after 6 months?

16. a. Estimation You invest $2,000 for 5 years at 4% compounded annually. Which increases your starting balance by the most?
 A. doubling the starting balance from $2,000 to $4,000
 B. doubling the interest rate to 8% annual interest
 C. doubling the time from 5 years to 10 years
 b. Calculator Check your answer to part (a). By how much did the balance increase?

Find the simple interest earned in each account.

17. $1,000 at 5% for 9 months   **18.** $4,500 at 4% for 8 months

19. $2,000 at 3.6% for 3 months   **20.** $500 at 1.75% for 180 days

Real-World Connection

Banks offer savings accounts that earn compound interest at a variety of rates.

The spreadsheet shows calculations using the compound interest formula. Use it for Exercises 21 and 22.

	A	B	C	D	E
1	Year	Balance at Start of Year	Rate	Interest	Balance at End of Year
2	1st	$3,000.00	0.04	$120.00	$3,120.00
3	2nd	$3,120.00	0.04	$124.80	$3,244.80
4	3rd	$3,244.80	0.04	$129.79	$3,374.59

21. State which column corresponds to each variable in the formula.
 a. p **b.** r **c.** t **d.** B

22. Show how to calculate the amount in cell E4.

23. Writing in Math Certificates of Deposit (CDs) allow you to save money at fixed compound interest rates, provided that you do not withdraw your money before the end of the term. What advantages do longer term CDs have? Disadvantages?

Certificate of Deposit	
Term	**Interest Rate**
3 month	2.72%
6 month	2.77%
12 month	2.96%
24 month	3.93%
36 month	3.64%
60 month	4.22%

LOANS CDs BILLS ACCOUNT

C Challenge

24. Reasoning You invest $2,000 in a simple interest account. The balance in 8 years is $2,720. What is the interest rate?

25. Stretch Your Thinking Three friends are guessing the number of buttons in a jar. They guess 495, 514, and 537. One guess is off by 8, another by 15, and another by 27. How many buttons are in the jar?
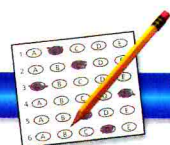
Test Prep

Multiple Choice

26. Which expression would you use to find the balance for $200 invested for 6 years at a 5% interest rate compounded annually?
A. $200(1.06)^5$ **B.** $200(0.06)(5)$ **C.** $200(1.05)^6$ **D.** $200(0.05)(6)$

27. You borrow $200 at a 3% simple interest rate. About how much interest will you owe in 18 months?
F. $6   **G.** $9   **H.** $12   **I.** $108

Short Response

28. You invest $100 for 5 years. Which will yield a greater balance, an account with a 3% simple interest rate or one with a 3% rate compounded annually? How much greater? Show your work.

Mixed Review

Algebra Solve each equation.

Lesson 2-6
29. $-3t - 10 = 11$ **30.** $\frac{1}{2}x - 5 = 7$ **31.** $\frac{p}{2} + 12 = 5$

Lesson 2-3
32. $5 + n = -4$ **33.** $-10 = x - 15$ **34.** $-12 + c = -1$

Checkpoint Quiz 2   Lessons 9-5 through 9-7

TEXT Instant self-check quiz online and on CD-ROM

1. Graph $y = 2x - 4$.   **2.** Graph $d = rt$ for a 35 mi/h rate.

3. You walk for 3 h, eat lunch for 1 h, bike for 1 h, and then do homework for 2 h. Sketch a graph that describes your speed.

4. Graph the total simple interest earned over 4 years on $850 invested at an annual rate of 4.4%.

5. If you save $850 at an interest rate of 3.5% compounded annually, how much money would you have in the account after 5 years?

9-8 Write an Equation

What You'll Learn

OBJECTIVE 1 To solve problems by writing equations

. . . And Why

To solve a problem using a function, as in Example 1

✔ **Check Skills You'll Need**   ❓ For help, go to Lesson 2-1.

Evaluate each expression using the values $r = 3, s = 4,$ and $t = -2.$

1. $4r$

2. $5s + t$

3. $-3r - s$

4. $\frac{1}{2}t + s$

5. $rs + t$

6. $2st - r$

OBJECTIVE

1

📱 **TEXT** Interactive lesson includes instant self-check, tutorials, and activities.

Solving Problems by Writing Equations

Writing a function rule in the form of an equation can help you solve a problem. Use variables for values that are unknown or subject to change.

1 EXAMPLE   **Real-World 🌎 Problem Solving**

Materials to make a toboggan cost $8 each. A craftsman budgets $2,000 this year and $3,000 next year. How many toboggans can he make each year?

Read and Understand   Your goal is to find the number of toboggans the craftsman can make each year using his budget and the cost of materials.

Plan and Solve   Write a function rule for the number of toboggans.

Words	amount budgeted	divided by	cost of materials	is the	number of toboggans

⬇

Let b = amount budgeted. Let n = number of toboggans.

Equation	b	$\div$	$\$8$	$=$	n

$$\frac{b}{8} = n$$

$$\frac{2,000}{8} = n \qquad \frac{3,000}{8} = n \quad \leftarrow \textbf{Substitute } b = 2,000 \textbf{ and } b = 3,000.$$

$$250 = n \qquad\quad 375 = n \quad \leftarrow \textbf{Solve for } n.$$

The craftsman can make 250 toboggans this year and 375 next year.

Look Back and Check   Multiply each number of toboggans by the cost.

$250 \cdot 8 = 2,000$ ✓ $375 \cdot 8 = 3,000$ ✓   ← **Both answers check.**

Real-World 🌎 Connection
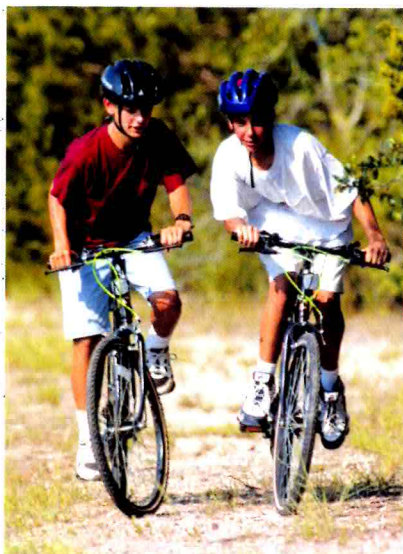
A toboggan is a wooden sled.

✔ **Check Understanding** **1** A 150-lb woman burns about 131 Cal/mi running 6 mi/h. She ran 40 mi this week and 35 mi last week. How many Calories did she burn each week?

② EXAMPLE **Real-World 🌐 Problem Solving**

Consumer Decisions You plan to purchase a mountain bike for $267 or a freestyle bike for $183. You have $90 and will save $7 each week. How long will it take to save enough for each bike?

Read and Understand Your goal is to find how many weeks it will take to save enough for each bike.

Plan and Solve Set up a function rule to represent the situation.

Words	total saved	is	savings	plus	amount saved each week	times	number of weeks

⬇

Let s = total amount saved. Let n = number of weeks.

Equation	s	=	$90	+	$7	·	n

Mountain bike

$267 = 90 + 7n$ ← Substitute 267 and 183 for s. →

$177 = 7n$ ← Solve for n. →

$25.3 \approx n$

Freestyle bike

$183 = 90 + 7n$

$93 = 7n$

$13.3 \approx n$

It will take 26 weeks to save enough for the mountain bike and 14 weeks for the freestyle bike.

Look Back and Check Use the number of weeks and the function rule to find how much was saved during each period.

$s = 90 + 7n$

$= 90 + 7(26) = 272$ ← The amount saved for the mountain bike.

$= 90 + 7(14) = 188$ ← The amount saved for the freestyle bike.

The mountain bike costs $267. The freestyle bike costs $183. You have saved enough to purchase either bike. Both answers are reasonable.

✔ **Check Understanding** ② Family membership at a science museum costs $89 per year. In addition, omnitheatre shows cost $22.50 per family each visit. How many omnitheatre shows could a family see if its yearly budget is $300?

Real-World 🌐 Connection

Mountain bikes are designed for rough terrain. Freestyle bikes are designed for roads.

EXERCISES

❓ For more practice, see *Extra Practice*.

Ⓐ **Practice by Example**

Examples 1, 2
(pages 503, 504)

Use a function rule to solve each problem. Check your answer.

🌐 **1. Travel** Suppose your family's car gets 23 mi/gal. Your family plans two trips. The first will be 540 mi and the second will be 270 mi. To the nearest gallon, how many gallons of gas will each trip require?
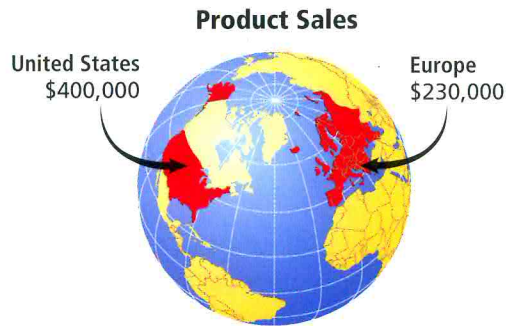
2. Sales A company's new product is sold for $16.50 per unit. About how many units does it sell altogether in the United States and Europe?

Product Sales

United States
$400,000

Europe
$230,000

3. Business The operating costs of a swimsuit distributor are $120,000 per year plus $23 per swimsuit. The current partners have $500,000 to spend and are considering a new partner who has $150,000. How many swimsuits can they produce with and without the new partner?

B **Apply Your Skills**

Use any strategy or a combination of strategies to solve each problem.

4. Saving You are saving $5 each week to attend a summer program that costs $300. You already have $84. You have also received a $50 scholarship for the program. How many weeks of saving do you need?

5. Measurement Using a balance scale and three weights, you can weigh any whole number of pounds from 1 lb to 13 lb. What are the weights?

Strategies

Draw a Diagram
Look for a Pattern
Make a Graph
Make an Organized List
Make a Table
Simulate a Problem
Solve a Simpler Problem
Try, Check, and Revise
Use Logical Reasoning
Work Backward
Write an Equation

6. Zoo The table below describes costs for admission to the zoo.
- **a.** What is the cost of admission for an adult? For a child?
- **b.** The cost of admission for a group of four people is $14. How many adults and how many children are in the group?
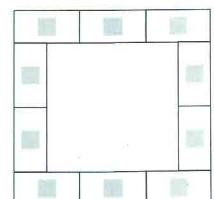
Children	Adults	Cost
0	4	$20
1	3	$18

7. Geometry How many diagonals does a regular 8-sided figure have?

8. Writing in Math During a sale, a store manager reduces a sweater's price by 30%. After the sale, the manager increases the price by 30%. Are the prices before and after the sale equal? Explain.

9. Five sand pails aligned side by side contain a total of 100 shells. Each bucket contains two shells fewer than the bucket to its left. How many shells are in each bucket?
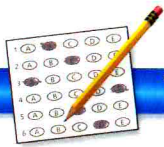
C **Challenge**

10. Stretch Your Thinking Copy the diagram at the right. Write the numbers 1 through 10 inside the rectangles so that the sum of the numbers along any side is 18.

Multiple Choice

11. Five pounds of potatoes cost $3.45. How much do 2 lb cost?

A. $.89   **B.** $1.38   **C.** $1.73   **D.** $2.14

12. On Monday $\frac{4}{7}$ of the students at school bought a hot lunch. About what percent of the students did NOT buy a hot lunch?

F. 37%   **G.** 43%   **H.** 57%   **I.** 62%

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13. All items in a store are 30% off. When you open a store charge card, you receive an additional 15% off your first purchase. Which expression represents the final price of an item that costs n dollars?

A. $0.3n - 0.15$ **B.** $0.7n - 0.85n$

C. $(0.7)(0.85)n$ **D.** $(0.7)(0.15)n$

Short Response

14. A survey of 1,500 users shows that 850 are extremely happy with an Internet service provider. If there are a total of 30,000 users, about how many are extremely happy? Show your work.

Mixed Review

Lesson 9-4

Algebra **Use the function rule $y = 3x - 3$. Find y for each input value.**

15. 3   **16.** 5   **17.** -4 **18.** $\frac{5}{6}$

19. 3.5   **20.** -7 **21.** -0.6 **22.** 0

23. $\frac{2}{3}$ **24.** 7.2   **25.** -2.8 **26.** $\frac{7}{6}$

Lesson 9-2

Write a rule for each arithmetic sequence. Then find the next three terms.

27. $15, 22, 29, 36, \ldots$ **28.** $9, 11.3, 13.6, 15.9, \ldots$ **29.** $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$

Math at Work

Artist

Artists use a variety of materials to create images—oils, watercolors, plaster, clay, or even computers. Graphic artists work for businesses. Fine artists display their works in galleries or museums.

Artists use angles and lines to draw in perspective, an important skill for any aspiring artist.

Take It to the NET For more information about artists, go to **www.PHSchool.com**.
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Transforming Formulas

What You'll Learn

OBJECTIVE
1 To solve for a variable

...And Why

To save for new clothes, as in Example 2

✓ Check Skills You'll Need

? For help, go to Lessons 2-3 and 2-4.

Solve each equation.

1. $a - 5 = 9$ **2.** $10 = -5x$ **3.** $12 = 4.5t$

4. $y + 8 = -12$ **5.** $\frac{p}{4} = -8$ **6.** $-3 + m = 1.2$

New Vocabulary • formula

OBJECTIVE

1 **Solving for a Variable**

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A **formula** is a rule that shows the relationship between two or more quantities. You can use the properties of equality to transform a formula and solve for a variable.

1 **EXAMPLE** **Transforming a Formula**

Reading Math

Transform and *formula* share the same root word, *form*.

Solve the formula for the perimeter of a rectangle $P = 2\ell + 2w$ for ℓ.

$$P = 2\ell + 2w \quad \leftarrow \text{Write the formula.}$$

$$P - 2w = 2\ell \quad \leftarrow \text{Subtraction Property of Equality}$$

$$\frac{P - 2w}{2} = \frac{2\ell}{2} \quad \leftarrow \text{Division Property of Equality}$$

$$\frac{P - 2w}{2} = \ell \quad \leftarrow \text{Simplify.}$$

Check for Reasonableness Use $P = 10$ ft, $\ell = 3$ ft, and $w = 2$ ft to check your answer.

$$P = 2\ell + 2w \qquad\qquad \ell = \frac{P - 2w}{2}$$

$$10 = 2(3) + 2(2) \qquad 3 = \frac{10 - 2(2)}{2} \quad \leftarrow \text{Substitute.}$$

$$10 = 10 ✔ \qquad\qquad 3 = 3 ✔$$

✓ **Check Understanding** **1** **a.** Solve $y = 2x - 4$ for x.

     b. Check your answer for part (a) using $y = 20$ and $x = 12$.

     c. **Reasoning** By what property do you notice that solving $y = 2(x - 2)$ for x will give the same result as your answer to part (a)?

You can transform formulas to solve real-world problems. First solve for the desired variable. Then substitute the values you know.

Savings Your bank account has an annual interest rate of 5.7%. You plan to withdraw the interest at the end of each year to buy new clothes. How much should you invest in order to have $100 to spend?

$$I = prt \quad \leftarrow \text{You plan to withdraw interest, so use the simple interest formula.}$$

$$\frac{I}{rt} = p \quad \leftarrow \text{Solve for } p.$$

$$\frac{100}{(0.057)(1)} = p \quad \leftarrow \text{Substitute 100 for } I, \text{ 0.057 for } r, \text{ and 1 for } t.$$

$$1{,}754.39 \approx p$$

🔵 You should invest about $1,755.

✔ **Check Understanding** **2** Find the interest rate that yields $120 interest each year on $2,000.

More Than One Way

Your scores on the first four math tests this year were 85, 98, 79, and 92. You need an average score of at least 90 for five tests to get an A in math. What minimum score do you need on the fifth test?

Will's Method

I can set up a formula for finding the mean and solve for the missing grade. I'll use a variable for each score. Let z = missing score.

$$\text{Formula for mean} \rightarrow \quad 90 = \frac{v + w + x + y + z}{5}$$

$$\text{Mult. Prop. of Equality} \rightarrow \quad 5(90) = v + w + x + y + z$$

$$5(90) - (v + w + x + y) = z \qquad \leftarrow \text{Subtraction Prop. of Equality}$$

$$5(90) - (85 + 98 + 79 + 92) = z = 96 \quad \leftarrow \text{Substitute and simplify.}$$

I need a minimum score of 96 on the fifth test.

Sarah's Method

I'll write an equation and work backward. The sum s of all the scores divided by the number of scores should be 90.

$$s \div 5 = 90 \quad \leftarrow \text{Write an equation.}$$

$$s = 450 \quad \leftarrow \text{Solve for } s.$$

The sum of the current scores is 85 + 98 + 79 + 92, or 354. So, for a total of 450, I need a score on the fifth test of 450 − 354, or 96.

Choose a Method

Your scores on three tests are 95, 89, and 75. You can replace your lowest score with the mean of the score and a retest. For an average score of 90, what is the minimum you must score on the retest?

Ⓐ Practice by Example

Example 1
(page 507)

Solve each equation for the variable in red.

1. $x = yz$

2. $t = \dfrac{u + v}{2}$

3. $p = 3r - 5$

4. $P = 4s$

5. $q = \dfrac{p}{r}$

6. $p = s - c$

7. $A = \frac{1}{2}bh$

8. $h = \dfrac{k}{j}$

9. $I = prt$

Example 2
(page 508)

10. How long would it take to earn $6,000 in interest on a principal of $9,000 at an annual simple interest rate of 4.1%?

11. You earn $1,400 simple interest on a principal of $12,500 in 4 years. What is the interest rate on the account?

12. Suppose you borrow money for a year at a simple interest rate of 7.2%. You pay $86.40 in interest. How much did you borrow?

Ⓑ Apply Your Skills

🌐 **13. a. Real Estate** A real estate agent earns 7% commission on the sale of a house. Write a formula for commission c. Let s = selling price.
　　b. The agent's commission is $8,400. Find the selling price of the house.

🌐 **14. Construction** Bricklayers use the formula $N = 7\ell h$ to estimate the number of bricks needed to cover a wall. N is the number of bricks, ℓ is the length of the wall in feet, and h is the height. If 980 bricks are used to build a wall 20 ft long, how high is the wall?

Solve each formula for the variable in red.

15. $V = \ell wh$

16. $r = \dfrac{d}{t}$

17. $F = \frac{9}{5}C + 32$

18. $V = \pi r^2 h$

19. **Writing in Math** Choose a formula from Exercises 15–18. Write and solve a problem involving the transformed version of the formula.

🌐 **20. Sales** You can use the formula $p = s - c$ to find the profit p with selling price s and cost c of an item. Find the selling price of an item that costs a seller $4.50 if she needs to make a profit of $.90.

Solve each equation for the variable in red.

21. $x = 3y + 6$

22. $t = \frac{1}{2}r$

23. $w = 3n + 5m$

24. $s = 0.5k - j$

25. $\frac{1}{2}b = d - 2$

26. $y = mx + b$

🌐 **27. Baseball** The formula for batting average a is $a = \dfrac{h}{n}$, where h is the number of hits and n is the number of times at bat. The highest major league lifetime batting average was .366 by Ty Cobb, who had 4,191 hits. About how many times at bat did he have?

Real-World 🌐 Connection

Careers Bricklayers are skilled in the craft of building walls, fireplaces, and other structures.

28. Geometry Find the radius of the circle. Use 3.14 for π.

29. Salary Weekly pay w is calculated using the formula $w = rh + 1.5rv$, where r is the regular hourly pay for a 40-h week, h is the number of regular hours you work, and v is the number of overtime hours you work beyond 40.
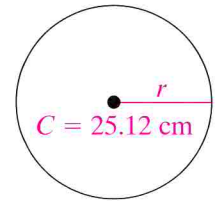
   a. Find the amount of your weekly paycheck for 40 h at a regular hourly pay of $9 per hour and 5 extra hours at the overtime rate.

   b. Find the number of extra hours you work at the overtime rate if the amount of your total weekly paycheck is $468.

$C = 25.12$ cm

C Challenge   Solve each equation for the variable in red.

30. $a^2 + b^2 = c^2$ **31.** $\frac{2}{3}m - 4 = p$

32. $V = \frac{1}{3}\pi r^2 h$ **33.** $A = \frac{1}{2}h(b_1 + b_2)$

34. Geometry The surface area S of a cube with side length e is $S = 6e^2$. Find the side length of a cube with a surface area of 150 cm^2.

35. Stretch Your Thinking What whole number, when added to 1,000,000, gives a greater result than 1,000,000 multiplied by the same number?

Test Prep

Multiple Choice

36. Solve for s in the equation $p = st$.

  A. $s = \frac{t}{p}$ **B.** $s = \frac{p}{t}$ **C.** $s = pt$ **D.** $s = p - t$

37. Solve for x in the function $y = 3x - 5$.

  F. $x = \frac{y + 3}{5}$ **G.** $x = \frac{y + 5}{3}$ **H.** $x = y + \frac{3}{5}$ **I.** $x = y + \frac{5}{3}$

38. Solve for a in the equation $\frac{1}{2}a - b = c$.

  A. $a = \frac{c + b}{2}$ **B.** $a = \frac{c - b}{2}$ **C.** $a = 2(c + b)$ **D.** $a = \frac{1}{2}bc$

Mixed Review

Lesson 8-6   **Find the missing side length. Round your answer to the nearest tenth of a unit.**

39.

8 m c 12 m

40.

25 in.   29 in. x
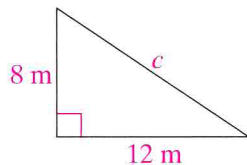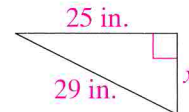
Lesson 7-2   **Classify each angle as *acute*, *obtuse*, *right*, or *straight*.**

41. $m\angle A = 55°$ **42.** $m\angle A = 180°$ **43.** $m\angle C = 99°$

Using estimation can help you find an answer, check an answer, or eliminate one or more answer choices.

1 EXAMPLE

A store is having a 30% off sale on all of its cross-training sneakers. What is the sale price on a pair of sneakers that costs $84.99?

A. $25.50   **B.** $51.99   **C.** $59.49   **D.** $79.99

You can estimate by changing $84.99 to a number that is easy to multiply in your head, such as $90.

A 30% discount will result in a sale price that is 70% of the regular price.

$s = 0.7c$ ← Write a function rule for sale price. Let s = sale price. Let c = regular cost.

$\approx 0.7(90)$ ← Substitute the estimated value.

$= 63$ ← Use mental math.

The sale price will be a little less than $63. The correct answer is C.

2 EXAMPLE

The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$. Sterling silver melts at approximately 893°C. What is the Fahrenheit temperature?

F. 998°F   **G.** 1,422°F   **H.** 1,639°F   **I.** 1,995°F

$F = \frac{9}{5}(893) + 32$ ← Substitute into the formula.

$\approx 2(900) + 32$ ← Estimate. $\frac{9}{5} \approx \frac{10}{5}$, or 2, and $893 \approx 900$.

$= 1,832$ ← This is an overestimate, since $900 > 893$ and $2 > \frac{9}{5}$.

According to your estimate, the answer is between choices H and I. So, you can eliminate choices F and G.

EXERCISES

1. A salon is offering a 20% discount on all hairstyles. What is the discount price for a style that regularly costs $23.50?

   A. $12.50   **B.** $14.75   **C.** $18.80   **D.** $22.00

2. The melting point of pure gold is 1,945°F. Convert this temperature to Celsius using the formula $C = \frac{5}{9}(F - 32)$.

   F. 1,063°C   **G.** 1,159°C   **H.** 1,205°C   **I.** 1,495°C

Chapter Review

Vocabulary

arithmetic sequence (p. 474)
balance (p. 500)
compound interest (p. 500)
conjecture (p. 475)

formula (p. 507)
function (p. 484)
geometric sequence (p. 475)
principal (p. 499)

sequence (p. 474)
simple interest (p. 499)

Reading Math: Understanding Vocabulary

Choose the correct term to complete each sentence.

1. A sequence is (arithmetic, geometric) if each term is found by adding the same number to the previous term.

2. A (formula, function) has only one output value for each input value.

3. (Balance, Principal) is an amount deposited or borrowed.

4. Interest paid on an original deposit and on any interest that has been left in an account is (compound, simple) interest.

5. A (conjecture, formula) is a prediction.

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Skills and Concepts

9-1 Objectives

▼ To make graphs
▼ To use graphs to make estimates

Graphs can help you visualize the relationship between data. Graphs have horizontal and vertical scales. Each scale is divided into intervals.

Graph the data in each table.

6.
Servings	1	2	3
Calories	280	560	840

7.
Time (days)	2	4	6
Pay ($)	15	30	45

8. Use your graph from Exercise 7 to estimate the pay for 11 work days.

9-2 and 9-3 Objectives

▼ To use arithmetic sequences
▼ To use geometric and other sequences
▼ To represent patterns in tables
▼ To write rules using tables

A **sequence** is a set of numbers that follow a pattern. You find each term of an **arithmetic sequence** by adding a common difference to the previous term, and of a **geometric sequence** by multiplying each term by a common ratio. Use a table to show a pattern, find a relationship, and find unknown quantities.

Identify each sequence as *arithmetic, geometric, both*, or *neither*. Write a rule for the sequence. Then find the next three terms.

9. $2, 10, 18, 26, \ldots$ 10. $48, 4, \frac{1}{3}, \ldots$ 11. $0, 1, 4, 13, 40, \ldots$

12. **a.** Make a table to represent the sequence $-6, -12, -18, -24, \ldots$.
    b. Write a variable expression for the rule. Then find the 100th term.

A **function** is a relationship that assigns exactly one output value for each input value. You can write a function rule by looking for patterns in a table. The graph of a function shows the relationship between inputs and outputs.
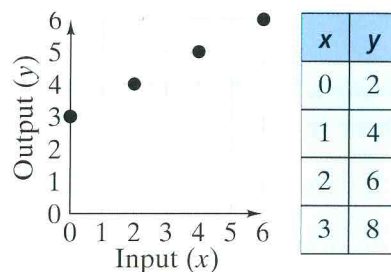
13. **a.** Make a function table for the graph.
    b. Write a function rule.

14. **a.** Write a rule for the function represented by the table.
    b. Graph the function.

15. Evaluate the function $y = -2x + 5$ for x-values of $-1, 0, 1, 2,$ and 3.

| x	y
0	2
1	4
2	6
3	8

9-6 Objectives

▼ To describe a graph

▼ To sketch a graph from a description

The graph of a function shows how one quantity changes relative to another. In a real-world context, you need to consider what is reasonable.

16. Describe a situation that the graph at the right might represent.

17. **Jogging** You jog at a rate of 3 mi/h. You jog for 3 h. You stop and spend 1 h eating lunch. Then you jog at 3 mi/h for an hour. Sketch a graph that represents the total distance you have traveled.
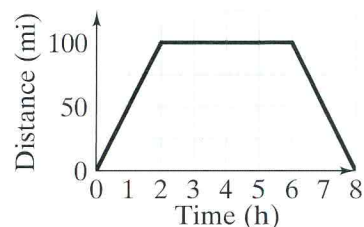
9-7 Objectives

▼ To find simple interest

▼ To find compound interest

You use the formula $I = prt$ to find **simple interest**. You use the formula $B = p(1 + r)^t$ to find the **balance** of an account with **compound interest**.

18. Explain the difference between simple interest and compound interest.

19. You deposit $1,500 in an account earning 6% simple interest. How much interest do you earn in five years?

20. You deposit $2,500 in an account that pays 5.7% interest compounded annually. What is the balance after five years?

9-8 and 9-9 Objectives

▼ To solve problems by writing equations

▼ To solve for a variable

Writing an equation or a function rule can help you solve a problem. A **formula** is a rule that shows the relationship between two or more quantities. Use properties of equality to solve for any variable in a formula.

21. **Fund-Raising** Suppose 250 people bought tickets to attend your school's annual world cultures festival. The international club spent $180 on preparations and made a $695 profit. How much did each ticket cost?

Solve each formula for x.

22. $z = 3x + y$

23. $k = -4xyz$

24. $\frac{1}{9}x - 4 = \frac{z}{3}$

Chapter 9

Chapter Test

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Write a rule for each sequence. Find the next three terms.

1. $1, 3, 9, 27, \ldots$ **2.** $4, 9, 14, 19, \ldots$

3. $3, 4, 6, 9, \ldots$ **4.** $10, 8, 6, 4, \ldots$

5. $-23, -19, -15, \ldots$ **6.** $6, 3, 1.5, 0.75, \ldots$

7. Identify each sequence in Exercises 1–6 as *arithmetic*, *geometric*, *both*, or *neither*.

8. a. Graph the data.
  b. Estimate the framed width for a photo that is 9.5 in. wide.
  c. Estimate the framed width for an 18-in. photo.

Picture Framing

Photo Width (in.)	Framed Width (in.)
5	4.17
8	6.67
12	10

Make a function table for each function.

9. the cost for 1 to 5 books at $2.95 each

10. the perimeter of a square with sides of 5, 6, 7, 8, and 9 in.

Write a function rule for each table.

11.

x	y
0	−2
1	−7
2	−12
3	−17

12.

x	y
0	1
1	3
2	5
3	7

13.

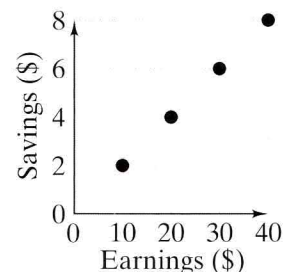| x	y
0	0
1	3
2	6
3	9

14. Graph the functions in Exercises 12 and 13.

Evaluate each function for $x = -2, 0,$ and 5.

**15.** $y = x - 5$ **16.** $y = 9 + x$

17. $y = 2x + 1$ **18.** $y = x^2 - 1$

19. Which is *not* an output for $y = 2x^2 - 5$?
  A. -3 **B.** 45   **C.** 27   **D.** -8

20. a. Make a function table for the graph.
  b. Write a rule for the function.
  c. How much is saved when $100 is earned?

21. Sports You dribble a basketball five times, pause briefly, and then shoot it into the hoop. Sketch a graph that describes the ball's height as a function of time.

22. Writing in Math Describe what the graph shows.

Oven Temperature
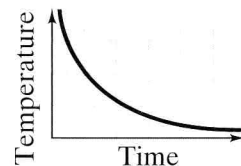
23. Suppose you borrow $500 from a bank that charges 14.5% interest compounded annually. What is the balance you owe after 4 years?

Solve each formula for n.

24. $3n - p = 6m$ **25.** $PV = nRT$

26. $s = (n - 2)180$ **27.** $(m + 1)n = b$

28. Home Repairs A plumber charges a customer $185 for repairs. The cost of parts is $65 and the plumber charges $30 per hour for labor. Write and solve an equation to find the number t of hours the plumber worked.

29. Science Density is found using the formula $D = \frac{m}{V}$, where m is mass in grams (g) and V is volume in cubic centimeters (cm^3). What is the volume of a pearl with a density of 2.72 g/cm^3 and a mass of 1.768 g?

Reading Comprehension Read each passage and answer the questions that follow.

Grade A To calculate grades for report cards, Ms. Sammler uses students' test scores during the semester. She also gives credit for class participation. First, she finds the mean test score for each student, which she calls "T." She then adds in the "P" factor— zero, three, or five points for class participation. She adds those items to get "G," the grade.

1. Hari's test scores are 80, 85, and 90. He never contributes in class, so he gets zero for participation. What will his grade "G" be?
   A. 83 **B.** 85 **C.** 88 **D.** 90

2. Ms. Sammler writes on the board and says, "Here is a mathematical equation that describes my system." What does she write?
   F. $G = \frac{1}{3}T + P$ **G.** $G = T + P$
   H. $3G = T + P$ **I.** $T = G + P$

3. Jennifer knows her test average is 88, so she is pleasantly surprised when she gets a 93 on her report card. What is her "P" factor?
   A. 0 **B.** 3 **C.** 5 **D.** 6

4. Jaime gets five points for class participation and receives an 85 on his report card. Which set could NOT have been his test scores?
   F. 60, 80, 100 **G.** 70, 70, 100
   H. 79, 80, 81 **I.** 80, 81, 83

Archaeology Archaeologists find the age of materials like bone, cloth, and wood using "Carbon-14 (C-14) dating." A tiny fraction (about one out of a trillion) of carbon atoms are radioactive and decay over time. Scientists measure the amount of C-14 left in an object to calculate its age. C-14 has a half-life of 5,700 years. This means that half of it remains after 5,700 years. In another 5,700 years, half of the remaining half will remain, and so on.

5. A 5,700-year-old bone has 10^{13} C-14 atoms. How many C-14 atoms did it have originally?
   A. 10^{13} **B.** 2×10^{13}
   C. 10^{14} **D.** 10^{26}

6. A wood fragment is about 11,000 years old. About what portion of its C-14 has decayed?
   F. 0.25 **G.** 0.50 **H.** 0.75 **I.** 1.0

7. How old would an object be if only $\frac{1}{8}$ of its original carbon-14 atoms remained?
   A. $2 \times 5,700$ years **B.** $3 \times 5,700$ years
   C. $4 \times 5,700$ years **D.** $8 \times 5,700$ years

8. Radioactive Potassium-40 (K-40) is found naturally in the human body. Its half-life is 1.3 billion years. After 1.3 billion years, how would the remaining percent of K-40 compare to the remaining percent of C-14?

   F. The percent of K-40 would be greater.
   G. The percent of C-14 would be greater.
   H. The same percent of each would remain.
   I. There would be more K-40 atoms than C-14 atoms.

Through the Ages

Applying Graphs Different animals have different life expectancies, which means they live different lengths of time. Reptiles, such as turtles, can live for more than a hundred years. Some insects, such as dragonflies, may live only a year or two.

You may know the saying that one dog year is like seven human years. That's because, on average, people live about seven times as long as dogs. But how do other animals compare? How old is a five-year-old cat or horse in human years? You can use graphs to make these comparisons.

The Long and the Short of It

Dragonflies can live as long as 6 or 7 years or as short as 6 or 7 months.

The front wings move independently from the back wings.

Ancient Insects

Archaeologists have found fossils of dragonflies over 200 million years old. Those dragonflies had wingspans up to 27 in. across. Today, the largest dragonflies have wingspans about 5 or 6 in. across.

Thorax

Head

Abdomen

Eyes

Six legs
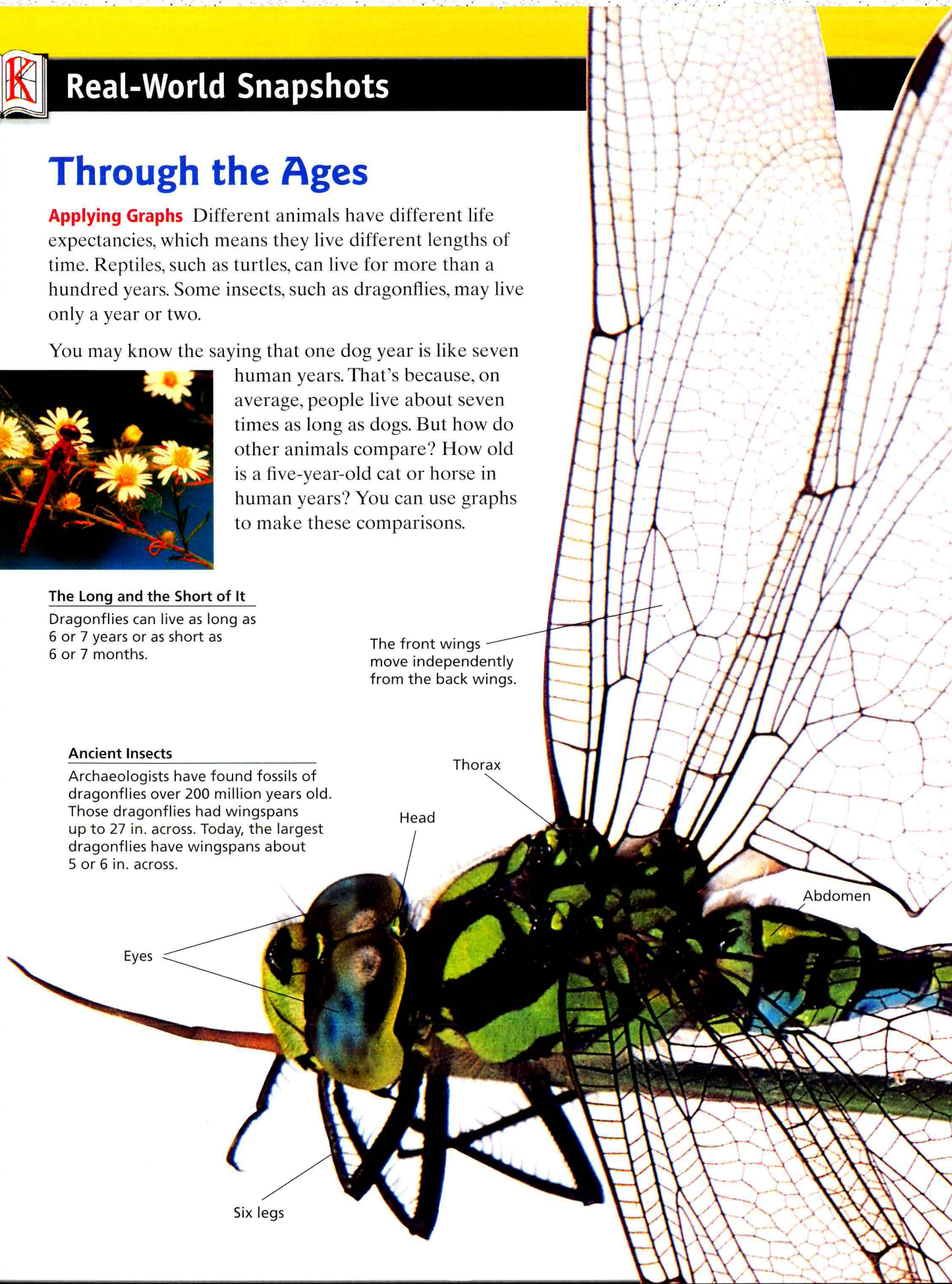
Put It All Together

Data File Use the information on these two pages and on page 467 to make a graph comparing animal and human life expectancies.

1. a. Start your graph by labeling the *x*-axis "Animal Age" and labeling the *y*-axis "Human Age." Use a scale up to 40 on the *x*-axis and 100 on the *y*-axis.

b. Graph the point (5, 35) to show that 5 dog years are equivalent to 35 human years. Draw a line through the origin and this point.

c. Use the line you graphed to find the "human age" of an 8-year-old dog.

d. Use the line you graphed to find your age in "dog years."

2. Use your graph from Question 1. Use 100 years as a human's maximum life span and choose at least three animals. Plot the points that compare each animal's maximum life span to a human's maximum life span. Use the points and the origin to draw lines for each animal.

3. Pick one of the animals from your graph. Compare the animal's life span to a human's life span. At what age would the animal be likely to start kindergarten? At what age would it graduate from high school? Mark those points on your graph.

4. Suppose you get a newborn kitten when you are 32 years old. How old will you be when you and the cat are the same age in human years?

5. Reasoning Many animals mature more quickly than people. They learn to walk a few hours after birth, and they are able to care for themselves in less than a year. Of the animals in your graph, which animal matures most rapidly compared to people? How can you tell by looking at the graph?

Take It to the NET For more information about dragonflies, go to **www.PHSchool.com**.
Web Code: abe-0953

Vibrant colors on the tail

Blade of grass

CHAPTER 10

Graphing in the Coordinate Plane

Lessons

Key Vocabulary

- angle of rotation (p. 560)
- coordinate plane (p. 521)
- image (p. 549)
- linear equation (p. 528)
- line symmetry (p. 554)
- nonlinear equation (p. 540)
- ordered pair (x, y) (p. 521)
- reflection (p. 555)
- rotation (p. 559)
- rotational symmetry (p. 559)
- slope (p. 533)
- transformation (p. 549)
- translation (p. 549)
- x-axis (p. 521)
- x-coordinate (p. 521)
- y-axis (p. 521)
- y-coordinate (p. 521)